

Math 2003

Test C

This part of the exam is to be done without a calculator

1. The smallest domain that is needed to show the entire graph of $f(x) = \sqrt{100 - x^2}$ on a graphing calculator is
- a) $-10 \leq x \leq 10$ b) $-5 \leq x \leq 5$ c) $0 \leq x \leq 10$
d) $0 \leq x \leq 5$ e) $-100 \leq x \leq 100$
2. The graph of $y = x^3 + 2$ is obtained from the graph of $y = x^3$ by
- a) translating vertically 2 units downward
b) translating vertically 2 units upward
c) translating horizontally 2 units to the right
d) translating horizontally 2 units to the left
e) translating both horizontally 2 units to the right and vertically 2 units up
3. A particle initially at $(0, 3)$ moves along a line of slope $m = 2$ to a new position (x, y) . Find y if $x = 5$.
- a) 5 b) 10 c) 13 d) 25 e) 28
4. Find the equations of ALL the asymptotes of $y = \frac{x}{x^2 + x + 1}$.
- a) $x = 0$ only b) $y = 0$ only c) $y = 0, x = -1, x = 1$
d) $y = 0, x = 0$ e) there are no asymptotes
5. $\lim_{x \rightarrow 5^+} \frac{x}{x-5} =$
- a) $-\infty$ b) ∞ c) 0 d) 5 e) The limit does not exist.
6. Let $f(x) = \begin{cases} x^3, & x \leq 2 \\ x-2, & x > 2 \end{cases}$ Evaluate $\lim_{x \rightarrow 2^+} f(x)$ if the limit exists.
- a) 8 b) 4 c) 0 d) 2 e) The limit does not exist

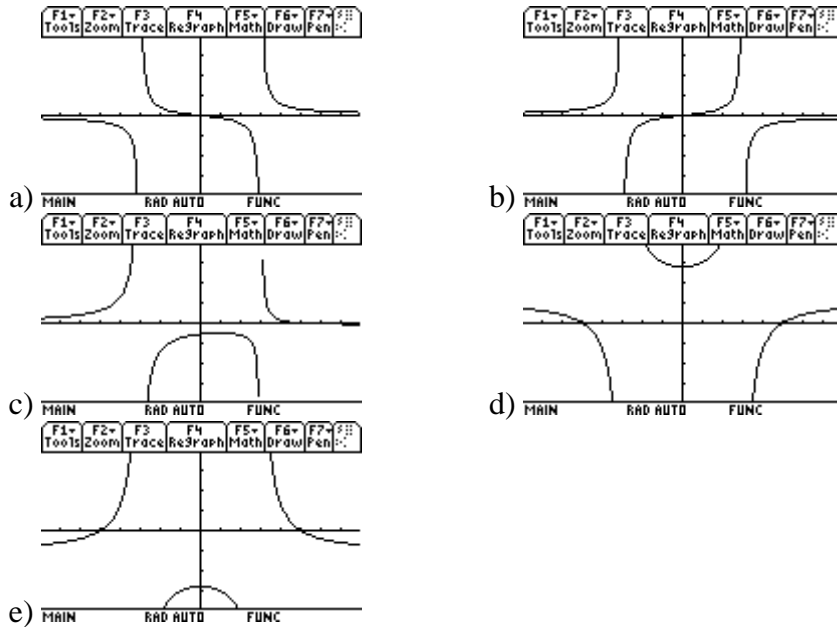
7. Find the limit: $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^2 + 1}$

- a) 0 b) ∞ c) -1 d) Does not exist e) -2

8. Find the vertical asymptote(s) of $f(x) = \frac{x-2}{x^2 + 3x - 10}$.

- a) $x = 2$ and $x = -5$ b) $x = -5$ only c) $x = 5$ only d) $x = 2$ only
 e) $y = 0$ only

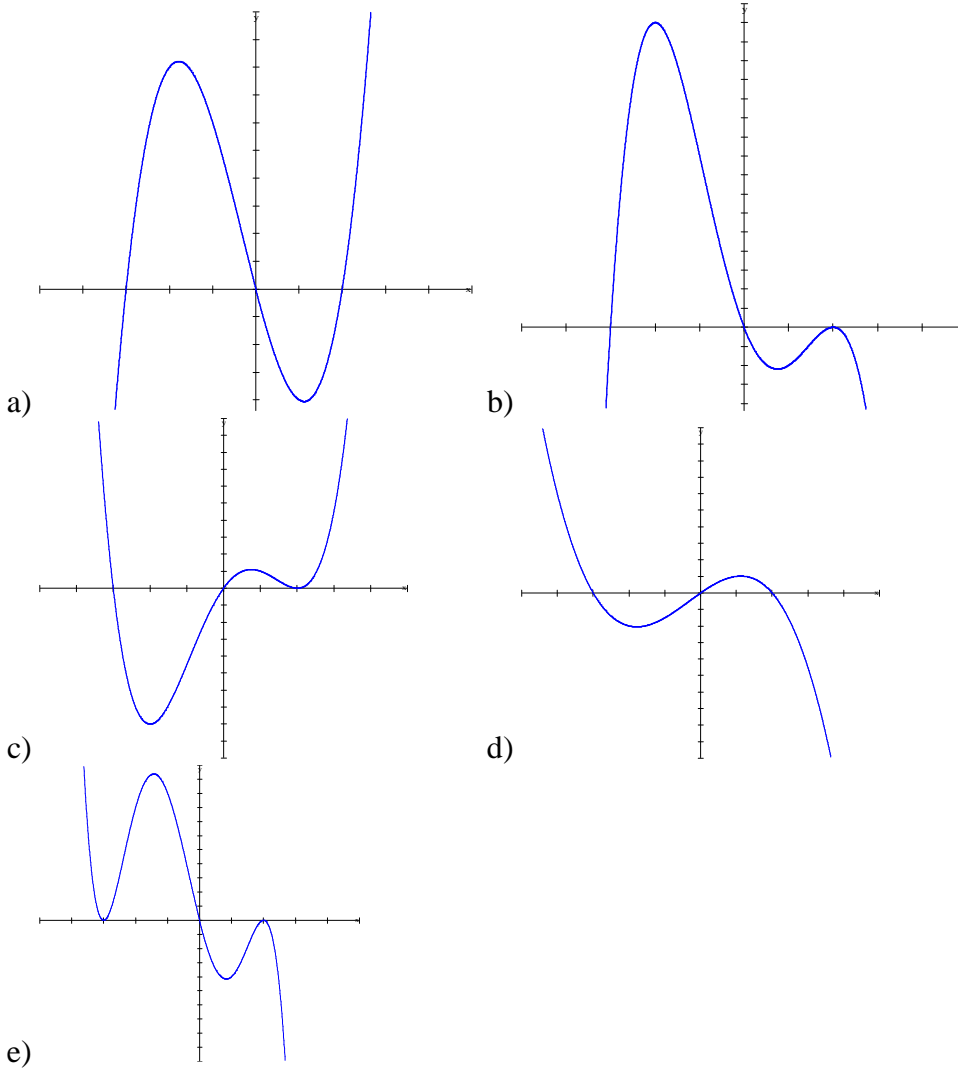
9. Which of the graphs below is the graph of the equation $y = \frac{x^2 - 25}{x^2 - 9}$?



10. Find the derivative of $f(x) = \frac{x^2}{3x^2 - 1}$

- a) $f'(x) = \frac{2x}{(3x^2 - 1)^2}$ b) $f'(x) = \frac{2x(6x^2 + 1)}{(3x^2 - 1)^2}$ c) $f'(x) = \frac{-2x}{(3x^2 - 1)^2}$
 d) $f'(x) = \frac{2x(6x^2 - 1)}{(3x^2 - 1)^2}$ d) $f'(x) = \frac{2(6x^2 - 1)}{(3x^2 - 1)^2}$

11. Which of the following is the graph of $f(x) = \frac{1}{4}x(x-2)^2(x+3)$?



12. Which choice best describes the behavior of the function $f(x) = \frac{x^2 - 3x + 4}{x - 2}$ at

$x = 2$?

- a) The function is continuous at $x = 2$
- b) The function has a hole at $x = 2$
- c) The function has a vertical asymptote at $x = 2$
- d) The function is discontinuous everywhere.

e) $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 4}{x - 2} = \infty$

13. Determine the average cost of producing 50 items if the total cost is given by the function $C(x) = 2x^2 - 200x + 100,000$ where x is the number of units produced.

- a) \$95,000 b) \$9,500 c) \$1,900
 d) \$190 e) Not enough information is given to determine the average cost.

14. The side of a square is increasing at the rate of 2 feet per minute. Find the rate at which the area is increasing when the side is 7 feet long.

- a) $28 \frac{ft^2}{min}$ b) $14 \frac{ft^2}{min}$ c) $49 \frac{ft^2}{min}$ d) $28\pi \frac{ft^2}{min}$ e) $49\pi \frac{ft^2}{min}$

15. Find the derivative of $f(x) = x\sqrt{3x-5}$ with respect to x .

- a) $\frac{1}{2}\sqrt{3x-5}$ b) $\frac{3}{\sqrt{3x-5}}$ c) $\frac{6x-5}{\sqrt{3x-5}}$ d) $\frac{7x-10}{2\sqrt{3x-5}}$
 e) $\frac{9x-10}{2\sqrt{3x-5}}$

16. If $f(x) = x^5 + x^3 - 2x + 1$, evaluate $f'(x)$ at $x = 1$.

- a) 0 b) 1 c) 5 d) 6 e) 7

17. The augmented matrix shown represents the solution of a system of equations in the variables (x, y, z, w) . What is its solution?

$$\begin{bmatrix} 1 & 0 & -7 & 0 & -3 \\ 0 & 1 & 8 & 0 & 10 \\ 0 & 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- a) $\left(-3, 10, \frac{5}{4}, 9\right)$ b) $\left(\frac{x+3}{7}, \frac{10-y}{8}, z, 9\right)$ c) $\left(\frac{z+3}{7}, \frac{10-z}{8}, 9, z\right)$
 d) $(-3+7z, 10-8z, z, 9)$ e) No solution

18. Given $A = \begin{pmatrix} 2 & -3 \\ 0 & \frac{1}{5} \end{pmatrix}$ and $B = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$, find A^2B .

- a) $\begin{bmatrix} 98 \\ 2 \\ 5 \end{bmatrix}$ b) $B = \begin{bmatrix} -26 \\ 2 \end{bmatrix}$ c) $\begin{bmatrix} 8 & 90 \\ 0 & \frac{2}{5} \end{bmatrix}$ d) $\begin{bmatrix} -58 \\ 2 \\ 5 \end{bmatrix}$ e) $\begin{bmatrix} 8 & 18 \\ 0 & \frac{2}{5} \end{bmatrix}$

19. If $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, find the inverse of A , A^{-1} .

a) $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

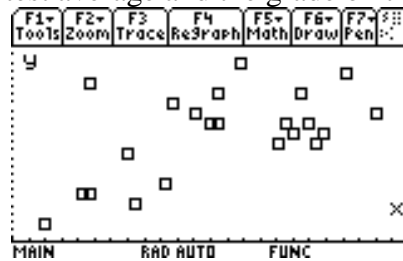
b) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$

c) $A^{-1} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{4} \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$

e) The inverse does not exist.

20. The scatter diagram below shows the relationship between the average on exams prior to the final exam, x , and the grade on the final exam, y , of a class of 24 students in 2003. Based upon the given data, the correlation coefficient showing the relationship between the test average and the grade on the final is approximately



- a) -0.98 b) -0.56 c) 0.05 d) 0.56 e) 0.98

21. Find the instantaneous rate of change of the volume of a cube with respect to a side of length x .

- a) x b) $3x$ c) $6x$ d) $3x^2$ e) x^3

22. Find the slope of the curve $4x^2y = 16$ at the point $(2, 1)$.

- a) -2 b) -1 c) $-\frac{1}{2}$ d) 1 e) $\frac{7}{4}$

23. The demand function and cost functions for x units of a product are

$p = \frac{60}{\sqrt{x}}$ and $C = 0.65x + 400$. Find the marginal profit when $x = 100$.

- a) \$2.35 per unit b) \$3.65 per unit c) \$4.58 per unit
d) \$187.35 per unit e) \$193.50 per unit

24. If the revenue function is given by $R(x) = 25x$ and the cost function is given

by $C(x) = 20x + 500$, the break even quantity will be:

- a) 40 b) 50 c) 100 d) 200 e) 1000

25. If the line $y - 3 = m(x - 1)$ is parallel to the line $3x + 2y - 5 = 0$, then $m =$

- a) $-\frac{3}{2}$ b) $-\frac{2}{3}$ c) $\frac{2}{3}$ d) $\frac{3}{2}$ e) 3

Test C

Some problems on this part of the exam may require the use of a calculator

26. Let g be the function defined by $g(x) = x^2 - x - 2$. Then $\frac{g(3+h) - g(3-h)}{2h} =$

- a) 0
- b) $2x-1$
- c) $\frac{5}{2}$
- d) 5
- e) $x - \frac{1}{2}$

27. Let f be the function defined by $f(x) = \frac{1}{4}x^4 + \frac{1}{2}x^2 + 1$. Find the x - and y -coordinates of the point on f for which the slope of f is -1 .

- a) $(-0.682, 1.287)$
- b) $(-0.682, 0)$
- c) The slope is never -1
- d) $\left(-1, \frac{7}{4}\right)$
- e) $(-0.453, 1.113)$

28. Find all points on the graph of $f(x) = 3x^4 - 52x^3 + 300x^2 - 672x + 900$ for which the tangent to f is horizontal.

- a) $(2, 388)$ only
- b) $(2, 0)$, $(4, 0)$ and $(7, 0)$
- c) $(2, 388)$, $(4, 452)$ and $(7, 263)$,
- d) $(4, 452)$ and $(7, 263)$ only
- e) $(4, 0)$ and $(7, 0)$ only

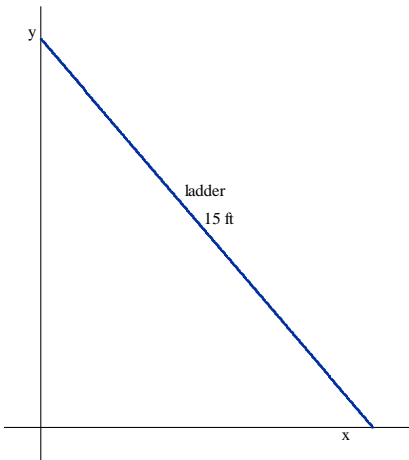
29. If $f(x) = \sqrt[5]{x^3 - 2x}$ then $f'(\sqrt{3}) =$

- a. 0.129
- b. 0.902
- c. 0.906
- d. 1.116
- e. 2.173

30. Find the equation of the tangent to $f(x) = x^{-x}$ at $x = 1$.

- a) $y = -x + 2$
- b) $y = x + 2$
- c) $y = -x - 2$
- d) $y = -x$
- e) $y = -x + 1$

31. A 15-foot ladder is leaning against a building as shown in the sketch below. The top of the ladder is at $(0, y)$ and the bottom is at $(x, 0)$. The ladder is falling because the ground is slippery; assume that $\frac{dy}{dt} = -12$ feet per second at the instant when $x = 9$ feet. Find $\frac{dx}{dt}$ at this instant.



- a) 6 feet per second
- b) 9 feet per second
- c) 12 feet per second
- d) 16 feet per second
- e) 20 feet per second

32. A bus will hold 60 people. The fare charged (p dollars) is related to the number x of people who use the bus by the formula $p = \left(3 - \frac{x}{40}\right)^2$. What number of people will make the marginal revenue equal to zero?
 a) 20 b) 40 c) 60 d) 80 e) 120

33. Solve the following system of equations:

$$a - 2b + 2c - 7d = 3$$

$$a - 2b + 3c + 9d = 3$$

$$3a - 6b + 7c + 23d = 8$$

- a) $\left(2 + \frac{45}{28}, b, \frac{4}{7}, -\frac{1}{28}\right)$, b) $\left(2b + \frac{45}{28}, \frac{4}{7}, b, -\frac{1}{28}\right)$
 c) $\left(\frac{45}{28}, -2, \frac{4}{7}, -\frac{1}{28}\right)$ d) $\left(2b + \frac{45}{28}, b, \frac{4}{7}, -\frac{1}{28}\right)$ e) no solution

34. The monthly cost of driving a car depends on the number of miles driven. Mary found that in January, it costs her \$395 to drive 700 miles and in July, it cost her \$420 to drive 800 miles. Express the monthly cost C as a function of the distance driven, d , assuming that a linear relationship gives a suitable model.

- a) $C = 0.25d + 220$
 b) $C = 220d + 0.25$
 c) $C = 220d - 0.25$
 d) $C = 0.25d - 220$
 e) $C = 0.125d + 230$

35. Let $f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = 2x + 3$. Find the domain of $f(g(x))$.

- a) $x \geq -\frac{3}{2}$ b) $x > 0$ c) $x < \frac{2}{3}$ d) $x > -\frac{3}{2}$

None of these

Answers to Test C

- | | | | | | | | | | |
|-----|---|-----|---|-----|---|-----|---|-----|---|
| 1) | a | 2) | b | 3) | c | 4) | b | 5) | b |
| 6) | c | 7) | a | 8) | b | 9) | d | 10) | c |
| 11) | c | 12) | c | 13) | c | 14) | a | 15) | e |
| 16) | d | 17) | d | 18) | d | 19) | d | 20) | d |
| 21) | d | 22) | b | 23) | a | 24) | c | 25) | a |
| 26) | d | 27) | a | 28) | c | 29) | b | 30) | a |
| 31) | d | 32) | b | 33) | d | 34) | a | 35) | d |