BARUCH COLLEGE

MATH 2207

FALL 2007

MANUAL FOR THE UNIFORM FINAL EXAMINATION

The final examination for Math 2207 will consist of two parts.

Part I: This part will consist of 25 questions.
No calculator will be allowed on this part.

Part II: This part will consist of 10 questions.
The graphing calculator is allowed on this part.

There may be a few new problem types on the exam that are not similar to the problem types that appear in this manual. If such problems appear, they will be similar to problems that you have seen during the semester.

GRADING: Each question will be worth 3 points.
Anyone who gets 34 or 35 questions correct will be assigned a grade of 100.
No points are subtracted for wrong answers.

CONTENTS OF THIS MANUAL:

One page showing the sample questions that correspond to each section of the current calculus textbook.
(When a section has been covered in class, the list indicates the problems that can be used in studying for the exam that includes that section during the semester.)
For each section, Part I type questions are listed first; then Part II questions are listed. Problems numbered 26 and higher (plus F25) are Part II problems.
TI-89 Facts for the Uniform Final Examination.
(This portion indicates the minimal calculator knowledge needed.)
Five sample final exams, A to E.
Additional sample problems.
Answers to the sample exams and additional problems.

Textbook Sections Corresponding to Sample Uniform Final Exam Questions
Fall 2007

Textbook: Larson and Edwards, Brief Calculus, an Applied Approach, Seventh Edition, Houghton Mifflin, 2006.

Section	Problems
1.5	B12, B29
1.6	A1, C23, D7, E16, F4
3.6	A2, A3, B13, B14, C24, C25, D8, E17, E18, F3, F5; D28, F25
2.1	A10, A11, B21, B22, C7, D15, D16, E25; C34, E33
2.2	A9, B20, C6, D14, E24, F1, F6; F26
2.3	A21, B8, C15, F26; A30, D33, E26, E27
2.4	A6, A7, B17, B18, C3, C4, D11, D12, E21, E22; F29
2.5	A8, B19, C5, D13, E23, F7
2.6	F27
2.7	A16, B2, B4, C10, D20, E5
2.8	A17, C11, D21, E7, F8
3.1	A12, A22, D4, E1, E12, F9; B26
3.2	A14, B11, B24, C8, C21, C22, D6, D18, E3; A26, C30, E32
3.3	A15, B3, B25, D19, E4, E6; A27, D30
3.4	A20, B7, B10, C14, D5, D24, E10
3.5	C17, D25, E11, E13; A33, C31
3.7	A32
3.8	A13, B9, B23, C15, D17, E2; C35
4.3	A5, B16, C2, D10, E20, F2, F10; A31, B31, C26, D29, D35
4.5	A4, B15, C1, D9, E19; D34, F28
5.1	A18, B6, E8
5.2 & 5.3	A29, D32
5.4	A19, C12, C13, D22, E3; B28, C28, E34
5.5	B5, D23; A28, B27, C27, D31, E35

TI-89 FACTS FOR THE UNIFORM FINAL EXAMINATION

The following information represents the minimal calculator usage students should be familiar with when they take the uniform final examination. Instructors are expected to provide much more information in class.

Limits can be evaluated with the TI-89 by using the limit function whose syntax is: **limit(*expression, variable, value*)**.
This function is found on the Home screen by using the F3 Calc key. The calculus menu that appears when F3 is pressed is shown in Figure 1. The limit function is selected by either pressing 3 or using the down arrow cursor to highlight choice 3:limit and then pressing ENTER.

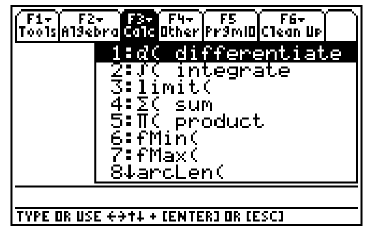


Figure 1

Example 1: Find $\lim_{x\to 16} \dfrac{x-4}{16-\sqrt{x}}$

Solution: Select the limit function as indicated above.
Complete the command line as follows and press ENTER:
limit((x-4)/(16-√(x)),x,16)
The answer is 1 as shown in Figure 2.

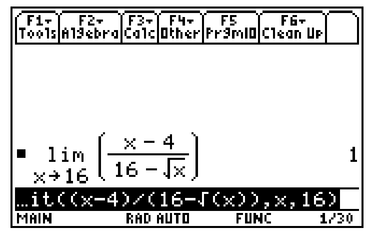


Figure 2

Example 2: Find $\lim_{\Delta x\to 0} \dfrac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x}$

Solution: In order to use the calculator for this problem it is best to think of Δx as a single letter such as t. That is, $\lim_{t\to 0} \dfrac{\frac{1}{x+t}-\frac{1}{x}}{t}$
Now enter on the command line the following: limit((1/(x+t)-1/x)/t,t,0).
The result is $-1/x^2$ as shown in Figure 3..
Note that example 2 could have been done without a calculator if the limit requested was recognized as the definition of the derivative of $f(x) = 1/x$ for which $f'(x) = -1/x^2$.

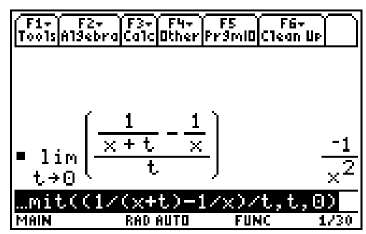


Figure 3

The syntax for the solve command and for finding the first or second derivative is:

solve(*equation, variable*)
***d*(*expression, variable*)** (first derivative, f ′(x), where f (x) = *expression*)
***d*(*expression, variable*, 2)** (second derivative, f ″(x), where f (x) = *expression*)

The solve command is obtained by pressing the F2 (Algebra menu) key in the Home Screen as shown in Figure 4 and then pressing ENTER to select choice 1:solve.

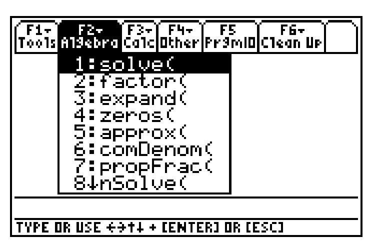


Figure 4

The differentiation operator *d* is choice 1 on the Home screen calculus menu shown in Figure 1. It can be more easily accessed by pressing the yellow 2nd key and then 8 (the *d* appears above the 8 in yellow). The purple D appearing above the comma key cannot be used for this purpose.

If an answer such as e^3 or 15/236 appears when a decimal answer is desired, the command can be repeated with the green diamond ◆ key pressed before pressing ENTER.

Example 3: The demand function for a product is $p = 10 - \ln(x)$, where x is the number of units of the product sold and p is the price in dollars. Find the value of x for which the marginal revenue is 0.

Solution: The revenue function is $R(x) = px = x(10 - \ln(x))$.
The marginal revenue is the derivative, $R'(x)$. This can be found by hand or by using the calculator command *d*(x*(10-ln(x)),x)
obtained by pressing the keys 2nd 8 (x * (1 0 - 2nd x x)) , x) ENTER
as shown in Figure 5. The result is $R'(x) = 9 - \ln(x)$.
To find the value of x for which the marginal revenue is 0 the equation $9 - \ln(x) = 0$ must be solved.

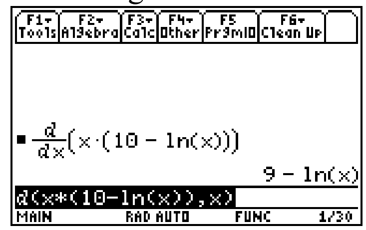


Figure 5

Press F2 (Algebra) ENTER as shown in Figure 4 and then complete the command line as: solve(9-ln(x)=0,x)
Pressing ENTER produces the result e^9. Since a decimal answer is desired, repeat the command by first pressing the green diamond ◆ key before pressing ENTER. The final answer is 8103.08 as shown in Figure 6 and can be rounded off to 8103.

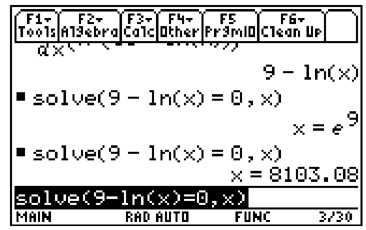


Figure 6

Example 4: The function $h(t) = \dfrac{30}{10 + e^{-0.1t+5}}$ has exactly one inflection point. Find it.

Solution: Recall that an inflection point occurs if $h''(t) = 0$ and the concavity changes sign. So first the second derivative is found with the command
d(30/(10+e^(-0.1t+5)),t,2)
where e^(is obtained by pressing ◆x (e^x appears in green over the x key).
Figure 7 is the result.

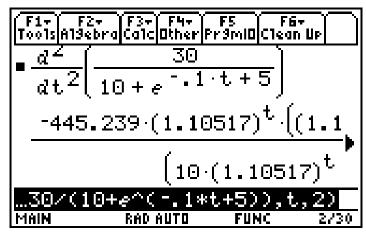


Figure 7

Recall that to see the entire result the up cursor arrow button must be pressed and then the right cursor arrow button must be pressed until the rest of the result on the right can be seen as shown in Figure 8. Thus,

$$h''(t) = \frac{-445.239(1.10517)^t((1.10517)^t - 14.8413)}{(10(1.10517)^t + e^5)^3}$$

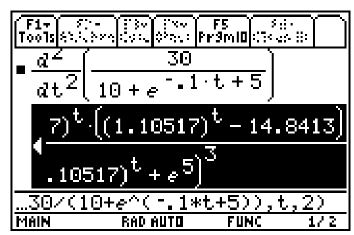


Figure 8

Move the cursor down to the command line again. The easiest way to solve $h''(t) = 0$ is as follows. First press F2 (Algebra) and select choice 1 so that the command line now only shows solve(
Press the up cursor arrow to highlight the expression for $h''(t)$ and press ENTER. The command line now shows the end of solve(*above expression*
Complete the command by entering =0,t)ENTER
The answer appears in Figure 9.

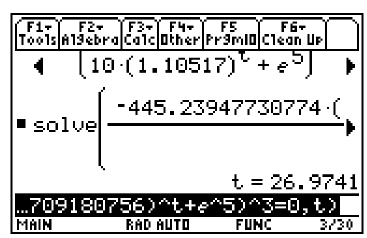


Figure 9

Since only one answer appears and it is known that an inflection point exists, there is no need to check to see if the concavity changes at $t = 26.9741$. It only remains to find the value of the original function at $t = 26.9741$. To do so, enter
30/(10+e^(-0.1*26.9741+5))
on the command line to obtain $h(26.9741) = 1.5$ as shown in Figure 10.
Therefore, the inflection point is (26.9741, 1.5).

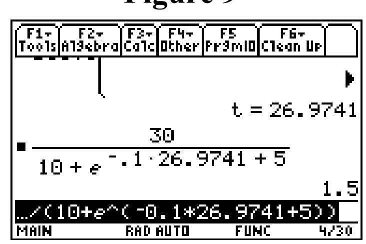


Figure 10

Example 5: 0.0559 and 1.788 are the only critical numbers for $f(x) = e^{5x} \ln(x/2)$. Determine if the critical point (0.0559, -4.73) is a relative minimum, a relative maximum or neither.

Solution 1: Recall that if $f''(0.0559)$ is positive the critical point is a relative minimum, if it is negative the point is a relative maximum, and if it is 0 the first derivative test must be used.

The key with the symbol | on it means "when" or "such that."

Now enter the following command:

d(e^(5x)*ln(x/2),x,2)|x=0.0559

Since $f''(0.0559) = -304.911$ is negative as shown in Figure 12, the critical point is a relative maximum.

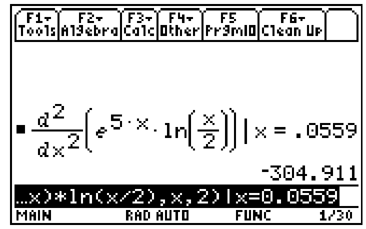


Figure 11

Solution 2: A graph that clearly showed the point (0.0559, -4.73) would reveal what was true for the critical point.

Press ◆F1 for the y= screen. Enter the function y1=e^(5x)*ln(x/2) as shown in Figure 12.

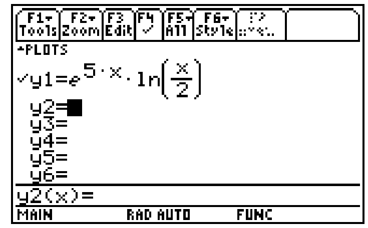


Figure 12

If F2 (Zoom) and choice 6:ZoomStd are selected (so that the *x* and *y* values go from -10 to 10), the result is Figure 13. Notice that this reveals nothing about the critical point in question. So it is desirable to look at the point more closely. (If you are "expert" at using ZoomIn, this approach can be used instead of the one shown. Just make sure the new center chosen has a *y* value near -4.73.)

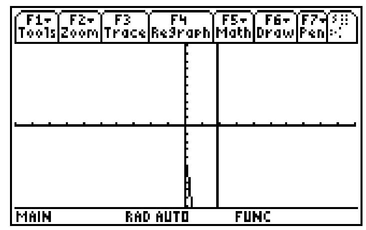


Figure 13

A good start might be to select values of *x* between -1 and 1 and values of *y* between -6 and -4. So press ◆F2 (Window) and enter the following values.

xmin=-1 xmax=1 xscl=1 ymin=-6 ymax=-4 yscl=1

as shown in Figure 14.

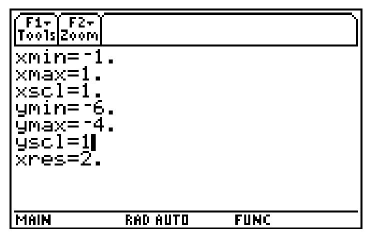


Figure 14

Then press ◆F3 (Graph). Figure 15 is the result.

Now press F3 (Trace) and then press the right cursor arrow a few times and observe values of *x* and *y* for each point on the graph shown. Clearly the high point is the critical point and therefore the critical point is a relative maximum.

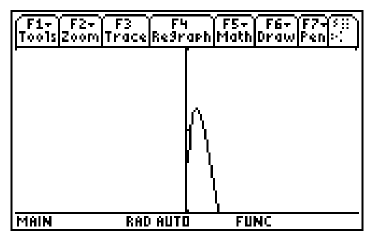


Figure 15

You should also be familiar with using F5 (Math) in the graph window to determine the exact location of a relative maximum or relative minimum displayed on the graph. For the graph shown in Figure 15 press F5 (Math) to obtain the menu shown in Figure 16.

Select choice 4:Maximum.

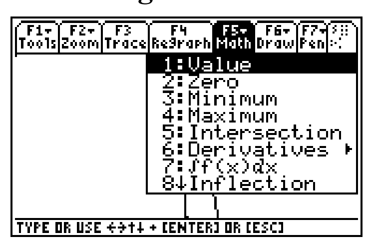


Figure 16

In response the "Lower Bound?" request, just use the cursor movement arrows to move the blinking cursor to the left of the maximum and press ENTER. Then, for the "Upper Bound?" request, move the blinking cursor to the right of the maximum and press ENTER. Figure 17 is the result and indicates that (0.05591, -4.73092) is a relative maximum.

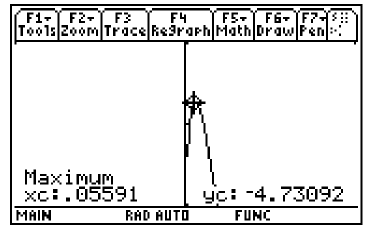


Figure 17

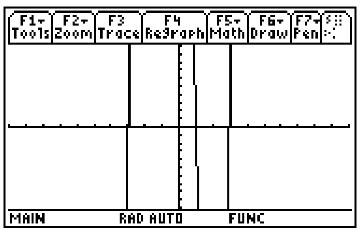
Example 6: Given $f(x) = x^4 + 14x^3 - 24x^2 - 126x + 135$

i) Find the critical numbers.

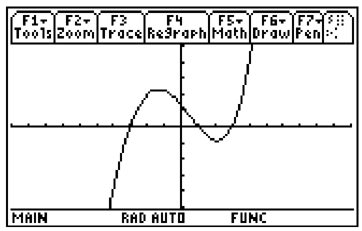
ii) Find the critical points

iii) All of the following are graphs of $f(x)$ in different graphical windows. Which graph most accurately portrays the function (shows its relative extrema, asymptotes, etc.)?

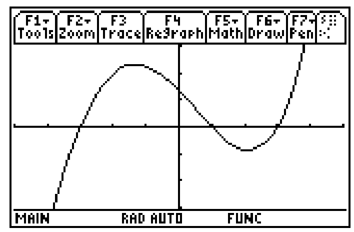
a)



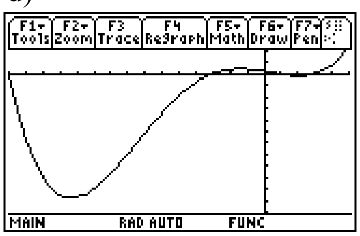
b)



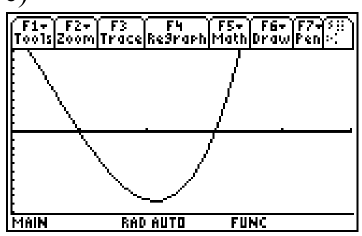
c)



d)



e)



Solution: i) The critical numbers are the solutions to
$0 = f'(x) = 4x^3 + 42x^2 - 48x - 126$.
Enter the command solve(4x^3+42x^2-48x-126=0,x) into the calculator as shown in Figure 18.
The critical numbers are x = -11.3145, -1.31026 or 2.12479

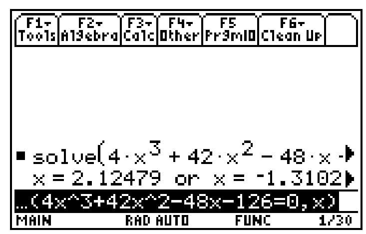


Figure 18

ii) For the critical points the value of y is needed for each of the critical numbers found.
First enter the function into y1: y1=x^4+14x^3-24x^2-126x+135.
Return to the Home screen by pressing the HOME key and press y1(-11.31450) ENTER in order to find $y = f(-11.31450) = -5401.64$. Repeat this for the other critical numbers. (Pressing the right cursor arrow removes the command line highlight and positions the cursor at the end of the line. Then the backspace key ← can be used to eliminate the -11.3145. Then enter the next critical number followed by ")".)
The critical points are (-11.3145, -5401.64), (-1.31026, 230.345) and (2.212479, -86.3943).

iii) A graphing screen should be used that includes the critical points found. The x values shown should include all values between -12 and 3. The y values shown should include all values between -5402 and 231. A reasonable window to choose would thus be

xmin=-15 xmax=5 xscl=1 ymin=-5500 ymax=500 yscl=500

The graph shown in Figure 19 is the result. So the answer is d.

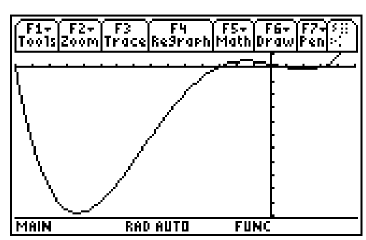


Figure 19

The integral sign appears in yellow over the key labeled 7. It can also be accessed in the home screen by pressing F3 Calc 2: ∫(integrate. The syntax is:

∫(4x^3,x) means $\int 4x^3 dx$ and returns x^4 (It is assumed that you will add the constant: x^4 + c.)

∫(4x^3,x,1,3) means $\int_1^3 4x^3 dx$ and returns 80.

Example 7: Find the area bounded between $y = x^4 - 8x^2$ and $y = 2x^2 - 9$.
Solution: We know the parabola opens upward and has (0, -9) as its vertex.
(If it is not clear to you as to why that is the vertex, notice $0 = y' = 2x \Rightarrow x = 0$.)
Enter the first function into y1: x^4-8x^2 so that it is available for graphing.
Find where its derivative is 0: solve(0=*d*(y1(x),x),x) ⇒ $x = -2, 0, 2$
Find the corresponding values of *y*: y1(-2) is -16, y1(0) is 0 and y1(2) is -16.
The critical points are (-2, -16), (0, 0) and (2, -16).
Enter the parabola in y2: 2x^2-9
Choose a graphing window that shows the vertex of y2 and the critical points of y1: xmin = -4 xmax = 4 xscl = 1 ymin = -20 ymax = 20 yscl = 5
Figure 20 is the result. Next we find the points of intersection.

solve(y1(x)=y2(x),x) ⇒ $x = -3, -1, 1, 3$

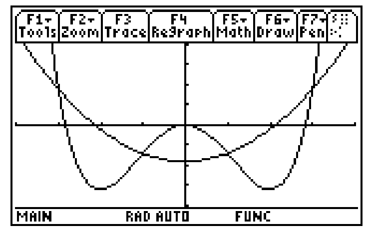


Figure 20

We can use either y1 or y2 to find the values of *y* for these 4 values of *x* (e.g. y1(-3) is 9). The 4 points of intersection are (-3, 9), (-1, -7), (1, -7) and (3, 9). There are no other points of intersection and these are the 4 points of intersection that appear in Figure 20.
So we wish to find the area shaded in Figure 21.
Notice that the parabola is on top between -3 and -1, the 4th degree function is on top between -1 and 1, and the parabola is on top between 1 and 3.

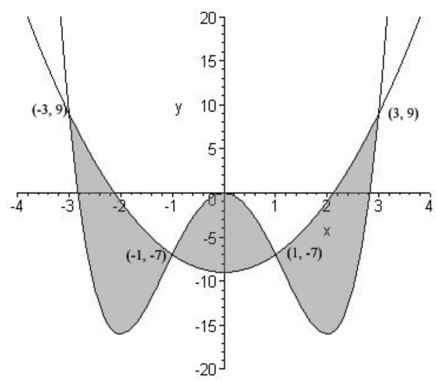


Figure 21

The area between -3 and -1 is

$$\int_{-3}^{-1}((2x^2 - 9) - (x^4 - 8x^2))dx = \int_{-3}^{-1}(-x^4 + 10x^2 - 9)dx$$

and this is ∫(-x^4+10x^2-9,x,-3,-1) = $\frac{304}{15}$

The area between –1 and 1 is

$$\int_{-1}^{1}((x^4 - 8x^2) - (2x^2 - 9))dx = \int_{-1}^{1}(x^4 - 10x^2 + 9)dx$$

and this is ∫(x^4-10x^2+9,x,-1,1) = $\frac{176}{15}$

The area between 1 and 3 is $\int_{1}^{3}((2x^2 - 9) - (x^4 - 8x^2))dx = \int_{1}^{3}(-x^4 + 10x^2 - 9)dx$

and this is ∫(-x^4+10x^2-9,x,1,3) = $\frac{304}{15}$

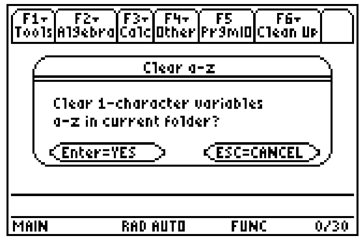
Therefore the desired area is $\frac{304}{15} + \frac{176}{15} + \frac{304}{15} = \frac{784}{15} = 52\frac{4}{15} = 52.2667$

MATRICES

Storing a matrix in a TI-89 calculator results in the calculator keeping that matrix in memory until it is erased. This is true whenever any result is stored in a variable. But from a practical point of view, this is usually only done for matrices. Hence, it is best if all variables are cleared after using the calculator with matrices. In the home screen, proceed as follows.

Press F6 (2nd F1)
The screen shown below appears.

Then press ENTER.
The screen shown below appears.



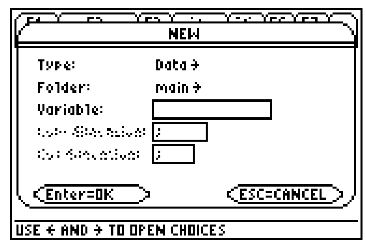
Press ENTER again. The home screen reappears.

The TI-89 will now be used to enter the matrix $A = \begin{bmatrix} 3 & 7 & 5 \\ 2 & 4 & -6 \end{bmatrix}$.

Press APPS

Press 6 (Data/Matrix Ed.)

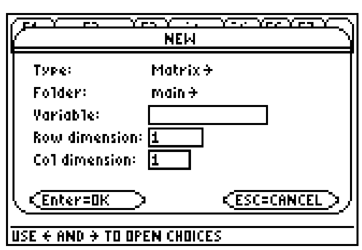
Press 3 (New)



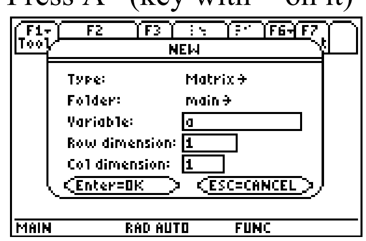
Notice that the word data is blinking. That indicates where the cursor is.

Press right cursor

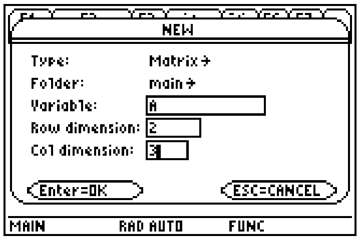
Press 2 (Matrix)



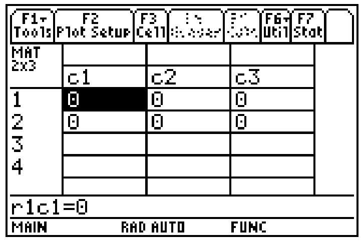
Press down cursor twice so that the cursor blinks in the variable rectangle.
Press A (key with = on it)



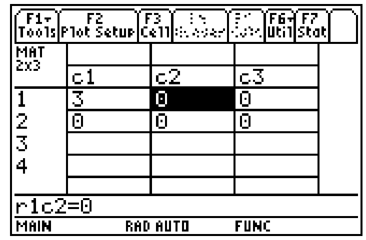
Press down cursor, 2,
down cursor, 3



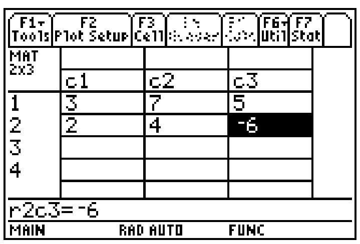
Press ENTER twice



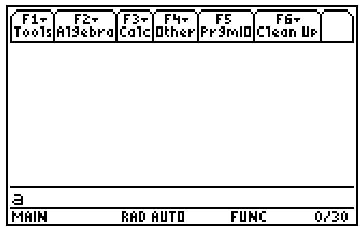
Press 3, ENTER



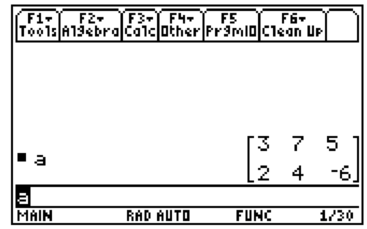
Press 7, ENTER
5, ENTER
2, ENTER
4, ENTER
Grey (-), 6, ENTER
(The cursor movement keys can be used to go back to any entry that needs changing.)



Press HOME
Press A (purple alpha, =)



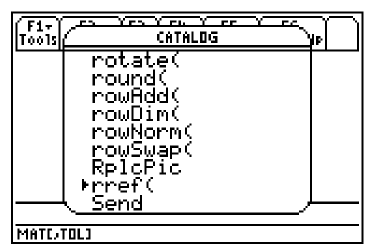
Press ENTER



Apart from going to the home screen, the last command (a, ENTER) was not needed. However, it does represent a useful way to display the final matrix and confirm the fact that the matrix was correctly entered into the calculator.

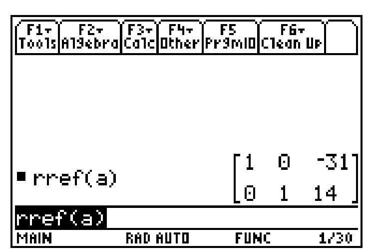
Example 8: Use the TI-89 to find the reduced row echelon form of $A = \begin{bmatrix} 3 & 7 & 5 \\ 2 & 4 & -6 \end{bmatrix}$, the matrix that was just entered

Solution: Matrix A was entered into the calculator above. Go to the Home screen and clear the command line. Next press the “catalog” key. Then press 2, which has the letter R in purple above it (when using catalog it is not necessary to press the alpha key before pressing a letter). Scroll down with the down cursor until you reach rref. Position the solid pointer so that it points to rref as shown in the figure on the right.



Press ENTER, alpha, a,), ENTER to obtain rref(a).
Your screen should look like the one shown on the right.

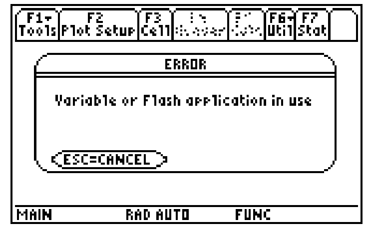
The reduced row echelon form of A is $\begin{bmatrix} 1 & 0 & -31 \\ 0 & 1 & 14 \end{bmatrix}$



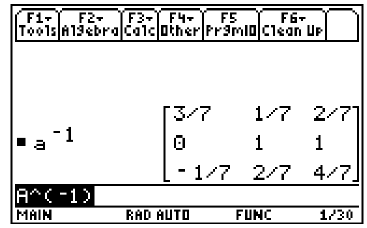
Note: All commands can be entered by hand if that is desired.
In this case pressing alpha, r, alpha, r, alpha, e, alpha, f, (, alpha, a,) would be used.

Example 9: Given the matrices $A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -3 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 9 & 5 \\ 1 & 4 & 3 \\ 3 & 14 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}$ find:

a) A^{-1} b) B^{-1} c) $2A - 3B$ d) $A^{-1}C$

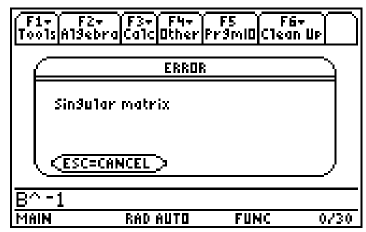


Solution: Enter the three matrices A, B and C. It is very likely that you encountered the screen shown on the right when you tried to enter A. That is because you must first delete the previous matrix entered that was also called A as was shown at the beginning of the section on matrices: F6 (i.e. 2nd F1), ENTER, ENTER.

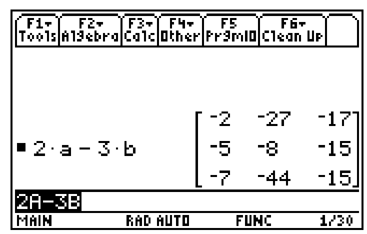


To find A^{-1}, simply enter A^(-1) on the command line of the home screen, i.e. alpha, ↑, =, ^, (, (-),), 1.

The result appears on the right. Hence, $A^{-1} = \begin{bmatrix} 3/7 & 1/7 & 2/7 \\ 0 & 1 & 1 \\ -1/7 & 2/7 & 4/7 \end{bmatrix}$

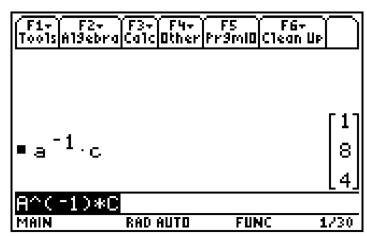


To find B^{-1} enter B^(-1) in the same way.
The figure shown on the right stating that B is a singular matrix is the result.
This means the matrix B does not have an inverse.



To find $2A - 3B$ simply enter that expression on the command line,
i.e. 2, alpha, ↑, =, -, 3, alpha, ↑, (, ENTER

Hence, $2A - 3B = \begin{bmatrix} -2 & -27 & -17 \\ -5 & -8 & -15 \\ -7 & -44 & -15 \end{bmatrix}$



To find $A^{-1}C$, enter that expression on the command line,
i.e. alpha, ↑, =, (, (-),) 1, *, alpha, ↑,), ENTER

Hence, $A^{-1}C = \begin{bmatrix} 1 \\ 8 \\ 4 \end{bmatrix}$

SAMPLE EXAM A

PART I: NO CALCULATOR ALLOWED

A1. Describe the intervals on which $f(x) = \begin{cases} 3 + x & \text{for } x \le 1 \\ x^2 + 2 & \text{for } x > 1 \end{cases}$ is continuous.

A) All real x B) All real $x \neq 3$ C) All real $x \neq 2$ D) All real $x \neq 1$ E) All x in the interval (0,4)

A2. Find $\lim_{x \to \infty} \dfrac{5x^4 + 1}{20x^4 + 3x^2 + 1}$

A) 1/4 B) 4 C) 0 D) 1 E) ∞

A3. The function $f(t) = \dfrac{t+1}{t^2 - 4t + 3}$ has vertical asymptotes

A) $t = 1$ only B) $t = -1$ only C) $t = 3$ only D) $t = 1$ and $t = 3$ only E) $t = -1$, $t = 1$ and $t = 3$

A4. Find the derivative of $y = 3\ ln(5x - 9)$.

A) $\dfrac{3}{ln(5x-9)}$ B) $\dfrac{3}{5x-9}$ C) $3\ ln(5)$ D) $\dfrac{15}{5x-9}$ E) $\dfrac{15}{ln(5x-9)}$

A5. If $f(x) = e^{x^2-3x}$, then $f'(x) =$

A) e^{x^2-3x} B) e^{2x-3} C) $e^{2x-3}(2x - 3)$ D) $e^{x^2-3x}(2x - 3)$ E) $e^{2x-3}(x^2 - 3x)$

A6. Find the derivative of $y = \dfrac{2x+3}{5x+7}$

A) $\dfrac{20x+29}{(5x+7)^2}$ B) $\dfrac{-1}{(5x+7)^2}$ C) $\dfrac{2}{5}$ D) $\dfrac{1}{(5x+7)^2}$ E) $\dfrac{-5(2x+3)}{(5x+7)^2}$

A7. Find the value of the derivative of $y = 5x(2x - 4)^4$ at $x = 3$.

A) 1040 B) 560 C) 320 D) 160 E) 960

A8. The derivative of $f(x) = \sqrt{1 - 3x^3}$ is

A) $\sqrt{1 - 9x^2}$ B) $-9x^2(1 - 3x^3)$ C) $\dfrac{-9x^2}{2\sqrt{1-3x^3}}$ D) $\dfrac{3x^3}{\sqrt{1-3x^3}}$ E) $-9x^2$

A9. Find the equation of the line tangent to the graph of $y = 2x^2 - 2x + 3$ at the point (1, 3).

A) $y = 2x - 2$ B) $y = 4x^2 - 6x + 5$ C) $y = 2x + 1$ D) $2x + y = 5$ E) $y = 2x + 2$

A10. Evaluate $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ for $f(x) = x^2 + 5x$.

A) $7x + \Delta x$ B) $3x + \Delta x$ C) $5 + \Delta x$ D) $2x + 5 + \Delta x$ E) $10x + \Delta x$

A11. Find $\lim\limits_{\Delta x \to 0} \dfrac{5x\Delta x + 2(\Delta x)^2}{\Delta x}$.

A) 0 B) $5x + 2$ C) $5x$ D) ∞ E) undefined

A12. Find the critical numbers for $g(x) = x^4 - 2x^2$.

A) $0, \pm\sqrt{2}$ B) $0, \pm 1$ C) $\pm\dfrac{\sqrt{3}}{3}$ D) only 0, -1 E) only 0

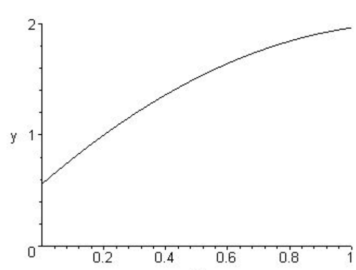
A13. Given $y = 3x^2$, find the value of the differential dy if x changes from 4 to 6.

A) 48 B) 24 C) 60 D) 12 E) 30

A14. (0, 5) (i.e. $x = 0$ and $y = 5$) is the only critical point of $f(x) = 2x^6 + 3x^4 + 5$. You do not have to verify this. Determine what is true of (0, 5).

A) (0, 5) is a relative maximum. B) (0, 5) is a relative minimum C) (0, 5) is a saddle point
D) (0, 5) is not a relative extremum E) no conclusion is possible

A15. On the interval $0 < x < 1$, for the graph shown,



A) $f'(x) > 0$ and $f''(x) > 0$
B) $f'(x) > 0$ and $f''(x) < 0$
C) $f'(x) < 0$ and $f''(x) > 0$
D) $f'(x) < 0$ and $f''(x) < 0$
E) None of the above

A16. Find dy/dx by using implicit differentiation if $xy + y = 5x^2$.

A) $\dfrac{9x-1}{x}$ B) $\dfrac{10x}{x+1}$ C) $\dfrac{10x-y}{x+1}$ D) $10x - 1$ E) $10x - y$

A17. At what rate is the area of a circle increasing if the radius is increasing at the rate of 10 feet per minute when the radius is 6 feet long? ($A = \pi r^2$)

A) 2π ft²/min B) 60 ft²/min C) 120π ft²/min D) 12π ft²/min E) 120 ft²/min

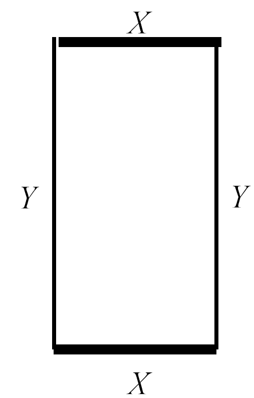
A18. If the marginal profit function is given by $P'(x) = 100 - 4x$ and the profit is \$200 when 10 units are produced, find the profit when 20 units are produced.

A) \$25 B) \$1200 C) \$400 D) \$1250 E) \$600

A19. Find the average value of $f(x) = \sqrt{x}$ on the interval [1, 4] (i.e. $1 \le x \le 4$).

A) 7/3 B) 7/2 C) 21/2 D) 14/9 E) 14/3

A20. Select the correct mathematical formulation of the following problem. A rectangular area must be enclosed as shown. The sides labeled x cost \$20 per foot. The sides labeled y cost \$10 per foot. If at most \$300 can be spent, what should x and y be to produce the largest area?



A) Maximize xy if $40x + 20y = 300$
B) Maximize xy if $20x + 10y = 300$
C) Maximize xy if $2x + 2y = 300$
D) Maximize $2x + 2y$ if $xy = 300$
E) Maximize $2x + 2y$ if $200xy = 300$

A21. What is the average rate of change of $f(x) = x^2 - 4x + 1$ on the interval [-1, 1]?

A) -4 B) 4 C) -6 D) -2 E) 2

A22. Find the open intervals on which $f(x) = \dfrac{x^2}{x^2+4}$ is decreasing.

A) $(-\infty, -2)$ and $(2, \infty)$ (i.e. $x < -2$ and $x > 2$) B) $(0, \infty)$ (i.e. $x > 0$) C) $(-2, 2)$ (i.e. $-2 < x < 2$
D) $(-\infty, 0)$ (i.e. $x < 0$) E) $(-\infty, \infty)$ (i.e. all real x)

A23. The system of linear equations on the right has as its solution

$$\begin{aligned} x + y - 3z &= -1 \\ y - z &= 0 \\ -x + 2y &= 1 \end{aligned}$$

A) only (1, 1, 1) B) only (-1, 0, 0) C) only (3, 2, 2) D) no solution E) $(-1 + 2z, z, z)$

A24. If $A = \begin{bmatrix} 1 & -2 & 4 \\ 5 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 7 & -1 \\ -2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & 4 \\ -1 & 3 \end{bmatrix}$, find $AB - 5C$.

A) $\begin{bmatrix} -23 & -17 \\ 15 & 0 \end{bmatrix}$ B) $\begin{bmatrix} -45 & 1 \\ 9 & 12 \end{bmatrix}$ C) does not exist because AB does not exist.
D) does not exist because AB and $5C$ have different dimensions.
E) does not exist because all three matrices must have the same dimensions for $AB - 5C$ to exist.

A25. When writing the system of equations shown in the form $AX = B$, what should A be?

$$\begin{aligned} 3x - y \quad &= 8 \\ -5x + 7y + 2z &= 4 \\ 32y + z &= 7 \end{aligned}$$

A) $\begin{bmatrix} 3 & -1 & 0 \\ -5 & 7 & 2 \\ 0 & 32 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 3 & -1 & 0 & 8 \\ -5 & 7 & 2 & 4 \\ 0 & 32 & 1 & 7 \end{bmatrix}$ C) $\begin{bmatrix} 3 & -1 & 8 \\ -5 & 7 & 2 \\ 32 & 1 & 7 \end{bmatrix}$ D) $\begin{bmatrix} 3 & 1 & 0 & 8 \\ -5 & 7 & 2 & 4 \\ 0 & 32 & -1 & 7 \end{bmatrix}$ E) $\begin{bmatrix} 8 \\ 4 \\ 7 \end{bmatrix}$

PART II: CALCULATOR ALLOWED

A26. The absolute maximum value of the function $f(x) = x^3 - 6x^2 + 15$ on the closed interval $[0, 3]$, (i.e. $0 \le x \le 3$) is

A) -7 B) -17 C) 4 D) -12 E) 15

A27. Find the inflection point for $g(t) = \dfrac{40}{5 + 2^{-t+4}}$

A) (4, 8) B) (1.678, 4) C) (0, 1.9048) D) (1.5546, 3.8290) E) (10.3782, 7.9808)

A28. Find the area of the bounded region between $f(x) = 4x^3 - 36x$ and $g(x) = 0$ (the x-axis).

A) 162 B) 112 C) 81 D) 34 E) 0

A29. Evaluate $\int_{-4}^{4} \sqrt{16 - x^2}\, dx$.

A) 0 B) $\dfrac{8\sqrt{6}}{3} = 6.53197$ C) $8\pi = 25.1327$ D) 32 E) Non-real result

A30. The cost in dollars of producing x units of a product is given by $C(x) = \dfrac{2x^2 - 2x + 9}{\sqrt{x}}$ for $x \ge 0$.

Determine the value of x when the marginal cost is 0.

A) 0.8 B) 1.4 C) 3.5 D) 4.2 E) 5.3

A31. Find the number of units to be produced in order to maximize revenue if the demand function is given by $p = 500 - \dfrac{500}{1 + 2^{-0.001x}}$.

A) 3462 B) 2154 C) 2.15427E-10 D) 1844 E) A maximum does not exist (false)

A32. What is the relative maximum of the function $y = f(x) = \dfrac{4x}{x^2 + 1}$?

A) (-1, 4) B) (1, 0) C) (0, 0) D) (1, 2) E) (2, 1.6)

A33. Find the number of units that will minimize the average cost function if the total cost function is

$$C(x) = \frac{x^2}{4} - 3x + 400$$

A) 6 B) 10 C) 20 D) 40 E) 80

A34. Solve the system of equations:

$$\begin{aligned} 3x - 4y - z &= 1 \\ 2x - 3y + z &= 1 \\ x - 2y + 3z &= 2 \end{aligned}$$

A) (0, 0, 1) B) $(x + 7z, y + 5z, z)$ C) $(x + 7z, y + 5z, 1)$ D) $(x - 7z, y - 5z, 1)$ E) no solution

A35. If $A = \begin{bmatrix} 1 & 5 \\ 2 & -3 \end{bmatrix}$, find A^{-1}

A) $\begin{bmatrix} 1 & 1/5 \\ 1/2 & -1/3 \end{bmatrix}$ B) $\begin{bmatrix} 3/13 & 5/13 \\ 2/13 & -1/13 \end{bmatrix}$ C) $\begin{bmatrix} -3 & 2 \\ 5 & -1 \end{bmatrix}$ D) $\begin{bmatrix} -1 & -5 \\ -2 & 3 \end{bmatrix}$ E) Does not exist

SAMPLE EXAM B

PART I: NO CALCULATOR ALLOWED

B1. If $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, find A^2.

A) $\begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 4 \\ 16 & 9 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 4 \\ 8 & -16 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 16 \\ 4 & 9 \end{bmatrix}$

B2. What is the slope of the line tangent to $xy^3 = 8$ at the point (1, 2)?

A) -2/3 B) 1 C) 12 D) 8 E) -8

B3. If $R = -x^3 + 3x^2 - 2$ is the revenue function, where x is the amount spent on advertising, then the point of diminishing returns (point of inflection) occurs when

A) $x = 1$ B) $x = -1$ C) $x = 6$ D) $x = -2$ E) Not enough information is given to decide

B4. If $xy = 3$, find dy/dt when $x = 6$ and $dx/dt = 4$.

A) 1/3 B) -1/3 C) 0 D) ½ E) undefined

B5. Find the area of the bounded region between $f(x) = 9 - x^2$ and $g(x) = 0$ (the x-axis).

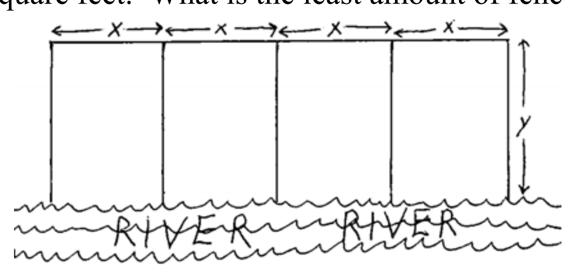
A) 0 B) 18 C) 36 D) 52/3 E) 92/3

B6. The marginal cost, in dollars, is given by $C'(x) = 2x + 200$. If the fixed overhead cost is \$2000, then the total cost of producing 10 items is

A) \$4500 B) \$2000 C) \$2100 D) \$3500 E) \$4100

B7. Select the correct mathematical formulation of the following problem.
A farmer wishes to construct 4 adjacent fields alongside a river as shown. Each field is x feet wide and y feet long. No fence is required along the river, so each field is fenced along 3 sides. The total area enclosed by all 4 fields combined is to be 800 square feet. What is the least amount of fence required?

A) Minimize xy if $4x + 5y = 800$
B) Minimize xy if $4x + 5y = 200$
C) Minimize $4x + 5y$ if $xy = 800$
D) Minimize $4x + 5y$ if $xy = 200$
E) Minimize $4(x + y)$ if $xy = 800$



B8. Find the average rate of change of the function $f(x) = -x^3 + 7x + 1$ on the closed interval [1, 4].

A) -14 B) 7 C) -6 D) -42 E) 4

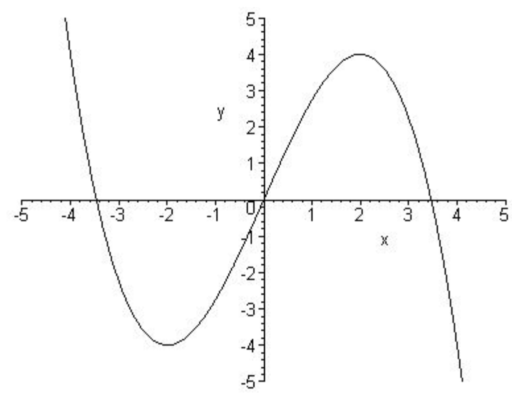
B9. Let the profit function for a particular item be $P(x) = -10x^2 + 160x - 100$. Use the marginal profit function to approximate the increase in profit when production is increased from 5 to 6.

A) 30 B) 40 C) 55 D) 60 E) 70

B10. The sum of a number and twice another number is 10. Find the largest possible product. (Notice that your answer is not one or both of the numbers, it is the product.)

A) 12.5 B) 8 C) 12 D) 12.6 E) 13

B11. The graph of $y = f(x)$ appears on the right. Estimate the points at which the absolute minimum and the absolute maximum occur on the interval [0, 3], that is, $0 \le x \le 3$.



A) absolute minimum: (0, 0); absolute maximum: (3, 2)
B) absolute minimum: (0, 0); absolute maximum: (2, 4)
C) absolute minimum: (-2, -4); absolute maximum: (2, 4)
D) absolute minimum: (4, -5); absolute maximum: (-4, 5)
E) absolute minimum: (3, 2); absolute maximum: (2, 4)

B12. Given

$$f(x) = \begin{cases} 2x + 3 & \text{for } x < 2 \\ x^2 & \text{for } x \ge 2 \end{cases}$$

find the values of the limits shown.

Choice	$\lim_{x \to 2^-} f(x)$	$\lim_{x \to 2^+} f(x)$	$\lim_{x \to 2} f(x)$
(A)	-1	4	undefined
(B)	2	2	2
(C)	3	0	undefined
(D)	7	4	undefined
(E)	7 and 4	7 and 4	7 and 4

B13. Find the horizontal asymptote of $f(x) = \dfrac{5x^3 + 3}{7x^3 - 4x^2 + 8}$

A) y = 5 B) y = 7 C) y = 0 D) y = 3/8 E) y = 5/7

B14. $f(x) = \dfrac{x - 2}{x^2 - x - 2}$ has as vertical asymptotes

A) $x = -1$ and $x = 2$ B) $x = 1$ only C) $x = 0$ only D) $x = -1$ only E) $x = 1$ and $x = -2$

B15. Find the derivative of $y = ln\,(2x + 3)^7$

A) $14\,ln\,(2x + 3)^6$ B) $14\,ln\,(2x + 3)^7$ C) $\dfrac{14(2x + 3)^6}{ln(2x + 3)^7}$ D) $\dfrac{14}{2x + 3}$ E) $\dfrac{7}{2x + 3}$

B16. Find the derivative of $f(x) = 3e^{-5x+7}$.

A) $-15e^{-5}$ B) $-15e^{-5x+7}$ C) $3e^{-5}$ D) $-15e^{-5x+7} + 3e^{-5x+7}$ E) $3e^{-5} + 3e^{-5x+7}$

B17. Find the derivative of $y = \dfrac{x^2}{3x^2 - 1}$

A) $\dfrac{2x}{(3x^2-1)^2}$ B) $\dfrac{2x(6x^2+1)}{(3x^2-1)^2}$ C) $\dfrac{-2x}{(3x^2-1)^2}$ D) $\dfrac{2x(6x^2-1)}{(3x^2-1)^2}$ E) $\dfrac{2(6x^2-1)}{(3x^2-1)^2}$

B18. If $g(x) = (x + 1)(x^2 - 3x + 1)$, then the slope of the line tangent to the graph of $g(x)$ at the point where $x = -2$ is

A) 0 B) 2 C) -3 D) -11 E) 18

B19. Find the slope of the line tangent to $y = \sqrt{3x + 10}$ at (2, 4).

A) 3 B) 1/8 C) 6 D) 2 E) 3/8

B20. Find the derivative of $f(x) = \dfrac{3}{\sqrt{x}}$.

A) $\dfrac{-3}{2\sqrt{x^3}}$ B) $\dfrac{-3\sqrt{x}}{2}$ C) $\dfrac{-6}{x^3}$ D) $\dfrac{6}{\sqrt{x}}$ E) $-6\sqrt{x}$

B21. Evaluate $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ for $f(x) = 2x^2 + 4x$.

A) 1 B) $\dfrac{\Delta x + 8x}{\Delta x}$ C) $4x + 4 + 2\Delta x$ D) $4 + 2\Delta x$ E) $12x + 2(\Delta x)^2 + 4\Delta x$

B22. Find $\lim\limits_{\Delta x \to 0} \dfrac{3x\Delta x + 5(\Delta x)^2 + 4\Delta x}{\Delta x}$.

A) ∞ B) $3x + 9$ C) $3x$ D) $3x + 4$ E) 0

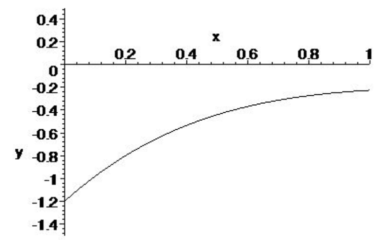
B23. If $y = x^2 + x$, find the value of the differential dy corresponding to a change in x of $dx = 0.1$ when $x = 2$.

A) 0.11 B) 0.5 C) 0.51 D) 1.2 E) 6.5

B24. (0, 7) (i.e. $x = 0$ and $y = 7$) is the only critical point of $f(x) = 7 - x^4 - 5x^6$. You do not have to verify this. Determine what is true of (0, 7).

A) (0, 7) is a relative maximum. B) (0, 7) is a relative minimum C) (0, 7) is a saddle point
D) (0, 7) is not a relative extremum E) no conclusion is possible

B25. On the interval $0 < x < 1$, for the graph shown,



A) $f(x) > 0$, $f'(x) < 0$ and $f''(x) > 0$
B) $f(x) < 0$, $f'(x) < 0$ and $f''(x) > 0$
C) $f(x) < 0$, $f'(x) > 0$ and $f''(x) < 0$
D) $f(x) < 0$, $f'(x) < 0$ and $f''(x) < 0$
E) $f(x) > 0$, $f'(x) > 0$ and $f''(x) < 0$

PART II: CALCULATOR ALLOWED

B26. One critical number of the function $y = x^4 - 2x^2 - 3x + 1$ is

A) 1.263 B) -0.148 C) 0.752 D) 1 E) -1.172

B27. Given the demand equation $p = -2x + 15$ and the supply equation $p = x + 3$, find the consumer surplus.

A) 20 B) 16 C) -8 D) 24 E) 44

B28. Find the average value of $f(x) = \dfrac{10}{4x+1}$ on the interval [1, 3] (i.e. $1 \le x \le 3$).

A) 2.38878 B) 6 C) 1.19439 D) -2 E) -1.25

B29. Evaluate $\lim\limits_{x \to 0} \dfrac{x^2}{(e^x - 1)^2}$

A) 1 B) 0 C) ∞ D) undefined E) e

B30. Find the interval(s) on which $f(x) = \dfrac{3 + x}{x^2 + 16}$ is increasing.

A) $-8 < x < 2$ B) $x < -8$ or $x > 2$ C) $-2 < x < 8$ D) $x < -2$ or $x > 8$ E) all real x

B31. The maximum value of $f(x) = xe^{-x}$ is

A) $e^{-1} \approx 0.368$ B) $e \approx 2.718$ C) 0.5 D) $-e \approx -2.718$ E) 0

B32. Solve the system of equations:

$$\begin{aligned} 2x - 3y + z &= 1 \\ -x - y \quad &= 4 \\ 3x \quad + 2z &= -3 \end{aligned}$$

A) (-19/7, -9/7, 18/7) B) (1, 1, 1) C) (-19k + 7, -9k + 7, 18k + 7) D) (-19, -9, 18) E) No solution

B33. Solve the system of equations:

$$\begin{aligned} x - 2y + 2z - 7u &= 3 \\ x - 2y + 3z + 9u &= 3 \\ 3x - 6y + 7z + 23u &= 8 \end{aligned}$$

A) (45/28, 0, 4/7, -1/28) only B) (2z + 45/28, 0, 4/7, -1/28) C) (45/28, -2, 4/7, -1/28) only
D) (2y + 45/28, y, 4/7, -1/28) E) no solution

B34. If $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, find the inverse of A, A^{-1}.

A) $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$ C) $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & -1/4 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/4 \end{bmatrix}$ E) Does not exist.

B35. Solve $\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 5 & -4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 4 \end{bmatrix}$

A) (1, 1, 1) B) (3, 9, -15) C) (6, 5, -3) D) (18, 15, -9) E) No solution

SAMPLE EXAM C

PART I: NO CALCULATOR ALLOWED

C1. Find the derivative of $f(x) = \ln\left(\dfrac{x^2-7}{x}\right)$.

A) $\dfrac{1}{x^2-7} - \dfrac{1}{x} = \dfrac{-x^2+x+7}{x(x^2-7)}$ B) $\dfrac{1}{x^2-7} + \dfrac{1}{x} = \dfrac{x^2+x-7}{x(x^2-7)}$ C) $\dfrac{2x}{x^2-7} + \dfrac{1}{x} = \dfrac{3x^2-7}{x(x^2-7)}$

D) $\dfrac{2x}{x^2-7} - \dfrac{1}{x} = \dfrac{x^2+7}{x(x^2-7)}$ E) $\dfrac{x}{x^2-7} + \dfrac{1}{x} = \dfrac{2x^2-7}{x(x^2-7)}$

C2. Find the equation of the line tangent to the graph of $y = xe^{2x-4}$ at the point where $x = 2$.

A) $y = e^2x + 2$ B) $y = 2x - 2$ C) $y = 3x - 4$ D) $y = 4x - 6$ E) $y = 5x - 8$

C3. For what value(s) of x is the derivative of $y = f(x) = \dfrac{x^2}{x^2-4}$ equal to zero or undefined?

A) $x = 0$ only B) $x = 2$ only C) $x = -2$ onlyD) $x = -2, 2$ only E) $x = -2, 0, 2$

C4. The derivative of $f(x) = x(4x - 7)^4$ is

A) $(4x - 7)^4$ B) $4x(4x - 7)^3$ C) $16x(4x - 7)^3$ D) $(20x - 7)(4x - 7)^3$ E) $17x(4x - 7)^3$

C5. The derivative of $f(x) = \sqrt{4x-3}$ is

A) $\dfrac{1}{2\sqrt{4x-3}}$ B) $\dfrac{2}{\sqrt{4x-3}}$ C) $(4x-3)^{1/2}$ D) $\dfrac{4}{\sqrt{4x-3}}$ E) $\dfrac{1}{2}(4x-3)^{1/2}$

C6. What is the <u>EQUATION</u> of the line tangent to $y = 4x^3$ at (2, 32)?

A) $y = 12x^2 - 16$ B) $y = 12x^2$ C) $y = 48$ D) $y = 12x + 8$ E) $y = 48x - 64$

C7. Given $f(x) = 7x^2$, find $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$.

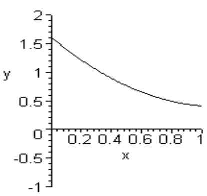
A) $14x + 7\Delta x$ B) $14x + (\Delta x)^2$ C) $7\Delta x$ D) 1 E) $14x + 7(\Delta x)^2$

C8. (0, 2) (i.e. $x = 0$ and $y = 2$) is the only critical point of $f(x) = x^5 + 3x^3 + 2$. You do not have to verify this. Determine what is true of (0, 2).

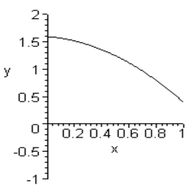
A) (0, 2) is a relative maximum. B) (0, 2) is a relative minimum C) (0, 2) is a saddle point
D) (0, 2) is not a relative extremum E) no conclusion is possible

C9. Given $f(x) > 0, f'(x) < 0$ and $f''(x) > 0$ on the interval $0 < x < 1$, pick the correct graph below.

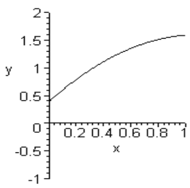
A)



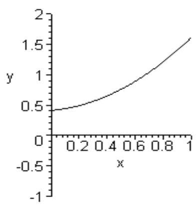
B)



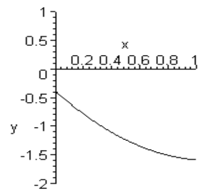
C)



D)



E)



C10. Use implicit differentiation to find dy/dx when $x = 1$ and $y = -2$ for $x^3 + 2xy + y^2 = 1$.

A) -1/2 B) -1 C) 0 D) 1/6 E) undefined

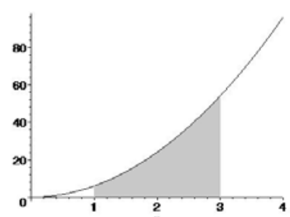
C11. The area of a square is increasing at the rate of 7 square inches per minute. At what rate is the length of one of the sides increasing at the moment when the side has length 4?

A) 7/4 B) 8 C) ½ D) 7/16 E) 7/8

C12. Evaluate $\int_0^4 (4x - 3\sqrt{x})dx$

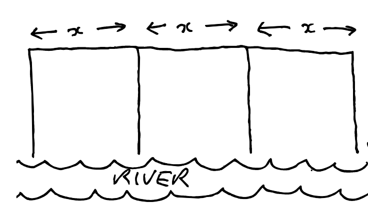
A) 28 B) 32 C) 4 D) 16 E) 10

C13. Find the area of the region shown where $y = 6x^2$.



A) 26 B) 156 C) 52 D) 48 E) 78

C14. A farmer wishes to construct 3 adjacent enclosures alongside a river as shown. Each enclosure is x feet wide and y feet long. No fence is required along the river, so each enclosure is fenced along 3 sides. The total enclosure area of all 3 enclosures combined is to be 900 square feet. What is the least amount of fence required?



A) 20 feet B) 120√2 feet C)120 feet D) 70√3 feet E) 360 feet

C15. The height in feet above the ground of an object traveling straight upwards is given by $s = 2t^3 + 4t$, where t is the time in seconds. What is the average velocity of the object between 0 and 3 seconds?

A) 58 B) 22 C) 31 D) 18 E) 33

C16. If $f(12) = 200$ and $f'(12) = -6$, estimate the value of $f(14)$ for the function $y = f(x)$.

A) 194 B) 188 C) 206 D) 214 E) 8

C17. Find the number of units to be produced in order to maximize revenue if the demand function is given by $p = 600x - 0.02x^2$ for $15{,}000 \leq x \leq 30{,}000$.

A) 15,000 B) 20,000 C) 25,000 D) 28,000 E) 30,000

C18. The reduced row echelon form that results from Gauss-Jordan row reduction of the system of linear equations shown appears below.

$$\begin{aligned} 2x - 3y + z &= -3 \\ x - 2y - z &= -9 \\ 3x - 4y + 3z &= 3 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 5 & 21 \\ 0 & 1 & 3 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find <u>all</u> solutions to the system of equations.

A) (21, 15, 0) B) $(5z + 21, 3z + 15, z)$ C) (1, 3, 4) D) $(21 - 5z, 15 - 3z, z)$ E) (-4, 0, 5)

C19. For the system of equations shown, what is the reduced row echelon form that results from Gauss-Jordan row reduction?

$$\begin{aligned} x - 2y + 2z + 7w &= 3 \\ x - 2y + 3z + 9w &= 3 \\ 3x - 6y + 7z + 23w &= 8 \end{aligned}$$

A) $\begin{bmatrix} 1 & -2 & 2 & 7 & 3 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 2 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 & -2 & 2 & 7 & 3 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 & -1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & -2 & 2 & 7 & 3 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$ E) $\begin{bmatrix} 1 & -2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

C20. Perform the following matrix operation: $2\begin{bmatrix} 3 & -5 \\ 2 & 0 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 3 & 4 \\ 0 & -1 \end{bmatrix}$.

A) $\begin{bmatrix} 8 & -11 \\ 1 & -4 \\ 8 & 7 \end{bmatrix}$ B) $\begin{bmatrix} 8 & -6 \\ 1 & -4 \\ 2 & 3 \end{bmatrix}$ C) $\begin{bmatrix} 4 & -11 \\ 1 & -4 \\ 8 & 5 \end{bmatrix}$ D) $\begin{bmatrix} 10 & -12 \\ -2 & -8 \\ 2 & 8 \end{bmatrix}$ E) $\begin{bmatrix} 8 & -9 \\ 1 & 4 \\ 8 & 5 \end{bmatrix}$

C21 and C22. For the function $f(x) = 4x - x^2 - 2$

C21. Find all critical points (x, y).

A) (2, 2) B) (2, -2) C) (2, 0) D) (-2, -14) E) (0, -2)

C22. Find the **absolute** extrema on the closed interval [0, 5] (i.e $0 \le x \le 5$).

A) minimum is -2 and maximum is 2 B) minimum is -7 and maximum is -2
C) minimum is -7 and maximum is 2 D) minimum is -7 and maximum is 5
E) minimum is -10 and maximum is 10

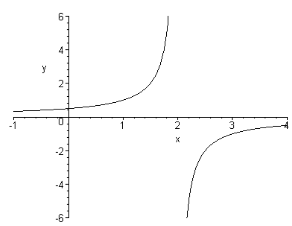
C23. In order to make the function $f(x) = \dfrac{x^2 - 1}{x - 1}$ continuous at $x = 1$ we must define

A) $f(1) = 0$ B) $f(1) = 1$ C) $f(1) = 2$ D) $f(1) = 1$ E) $f(1) = x$

C24. Find $\lim\limits_{x \to \infty} \dfrac{3x^2 + 5x - 7}{2x^3 + 4x + 2}$.

A) 1/8 B) 3/2 C) ∞ D) -7/2 E) 0

C25. $x = 2$ is a vertical asymptote of $y = f(x)$, whose graph appears on the right. Find $\lim\limits_{x \to 2^-} f(x)$



A) 0.4 B) 2 C) 0 D) ∞ E) - ∞

PART II: CALCULATOR ALLOWED

C26. The function $f(x) = 8e^{-0.1x^2}$ has exactly two points of inflection. The value of x for one of these points of inflection is

A) 8 B) -16 C) -0.8 D) 0 E) $\sqrt{5} \approx 2.236$

C27. Find the area of the region bounded by $y = 2x^2$ and $y = -x^2 + 12$.

A) 160/3 B) 32 C) 55.4256 D) 110.851 E) - 32

C28. Find the average value of $f(x) = e^{2x}$ on the interval [0, 2] (i.e. $0 \le x \le 2$).

A) 53.5982 B) 26.7991 C) 13.3995 D) 26.7991 E) 201.51

C29. For what open intervals is $f(x) = x^3 + 30x^2 + 5$ increasing?

A) $x < -20$ or $x > 0$ B) $x > 0$ only C) $-20 < x < 0$ D) $x < -20$ only E) $x < 0$ only

C30. Find the value of the relative maximum for the function $f(x) = x^3 - 15x$.

A) $-\sqrt{15}$ = -3.87 B) $\sqrt{15}$ = 3.87 C) $-10\sqrt{5}$ = -22.36 D) $10\sqrt{5}$ = 22.36 E) 12

C31. Given the total cost function $C(x) = 0.03x^2 + 32x + 4300$, find the value of x for which the average cost is a minimum.

A) -533.333 B) 533.333 C) 378.594 D) -378.594 E) 500

C32 and C33. Given $C = \begin{bmatrix} 2 & 6 \\ 3 & 8 \end{bmatrix}$ and $D = \begin{bmatrix} 6 & 2 \\ 5 & 7 \end{bmatrix}$

C32. Find CD^{-1}.

A) $\begin{bmatrix} \frac{23}{15} & \frac{13}{7} \\ \frac{21}{10} & \frac{37}{14} \end{bmatrix}$ B) $\begin{bmatrix} -9 & 13 \\ 4 & -4 \end{bmatrix}$ C) $\begin{bmatrix} -21 & 16 \\ \frac{-19}{2} & 8 \end{bmatrix}$ D) $\begin{bmatrix} \frac{1}{4} & \frac{13}{16} \\ \frac{1}{4} & \frac{9}{16} \end{bmatrix}$ E) $\begin{bmatrix} \frac{-1}{2} & 1 \\ \frac{-19}{32} & \frac{21}{16} \end{bmatrix}$

C33. Solve $CX = D$.

A) $\begin{bmatrix} \frac{23}{15} & \frac{13}{7} \\ \frac{21}{10} & \frac{37}{14} \end{bmatrix}$ B) $\begin{bmatrix} -9 & 13 \\ 4 & -4 \end{bmatrix}$ C) $\begin{bmatrix} -21 & 16 \\ \frac{-19}{2} & 8 \end{bmatrix}$ D) $\begin{bmatrix} \frac{1}{4} & \frac{13}{16} \\ \frac{1}{4} & \frac{9}{16} \end{bmatrix}$ E) $\begin{bmatrix} \frac{-1}{2} & 1 \\ \frac{-19}{32} & \frac{21}{16} \end{bmatrix}$

C34. Find $\lim_{\Delta x \to 0} \dfrac{\sqrt{(x+\Delta x)+3} - \sqrt{x+3}}{\Delta x}$

A) 0 B) Does not exist (undefined) C) $x + 3$ D) $\sqrt{x+3}$ E) $\dfrac{1}{2\sqrt{x+3}}$

C35. The side of a square is measured and found to be 6.78 inches long with a possible error of 0.03 inches. Use a differential to estimate the possible error that might result when computing the area of the square.

A) 0.0009 B) 1.37905 C) 0.4068 D) 0.03 E) 0.2034

SAMPLE EXAM D

PART I: NO CALCULATOR ALLOWED

D1. Solve the system of equations.

$$\begin{aligned} x \quad + \quad z &= 3 \\ x + y - 2z &= 10 \\ y - 3z &= 8 \end{aligned}$$

A) (3, 7, 1) B) (3 - z, 7 + 3z, 0) C) (3 - z, 7 + 3z, z) D) (3 - z, 7 + 3z, 1) E) No solution exists

D2. Perform the following matrix operation:

$$\begin{bmatrix} 3 & -4 & 2 \end{bmatrix} \begin{bmatrix} 5 & 0 & -1 \\ 2 & 4 & 3 \\ -2 & 1 & 6 \end{bmatrix}$$

A) $\begin{bmatrix} 3 & -14 & -3 \end{bmatrix}$ B) $\begin{bmatrix} 15 & 0 & 4 \\ 6 & -16 & 6 \\ -4 & 2 & 12 \end{bmatrix}$ C) $\begin{bmatrix} 3 \\ -14 \\ -3 \end{bmatrix}$ D) $\begin{bmatrix} 15 & 0 & -2 \\ 6 & -16 & 6 \\ -6 & -4 & 12 \end{bmatrix}$ E) Undefined

D3. Evaluate $\displaystyle \mathop{Lim}_{x \to 1} \frac{x-1}{3x^2 - 2x + 1}$.

A) 0 B) -1 C) 1/3 D) 1/4 E) Undefined

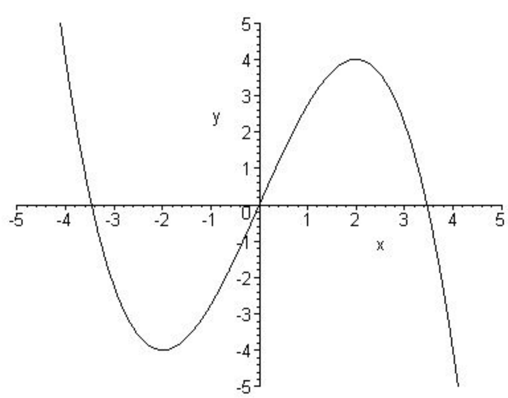
D4. Find the open intervals on which $f(x) = x^3 - 3x + 7$ is increasing.

A) (-∞, -1) or (1, ∞) (i.e. $x < -1$ or $x > 1$) B) (1, ∞) only (i.e $x > 1$ only)
C) (-1, 1) (i.e. $-1 < x < 1$) D) (- ∞, -1) only (i.e. $x < -1$ only)
E) (-∞, ∞) (i.e. all real x)

D5. Of all numbers (positive, negative and zero) whose difference is 4, find the two that have the minimum product. One of the numbers is:

A) 4 B) -3 C) -2 D) 1 E) -1

D6. The graph of $y = f(x)$ appears on the right. Estimate the points at which the absolute minimum and the absolute maximum occur on the interval [-3, 0], that is, $-3 \le x \le 0$.



A) absolute minimum: (-2, -4); absolute maximum: (2, 4)
B) absolute minimum: (0, 0); absolute maximum: (2, 4)
C) absolute minimum: (-2, -4); absolute maximum: (0, 0)
D) absolute minimum: (4, -5); absolute maximum: (-4, 5)
E) absolute minimum: (-3, -2); absolute maximum: (0, 0)

D7. The function defined by $f(x)=\begin{cases} x^2-9 & \text{if } x \le 4 \\ 3x-5 & \text{if } 4 < x < 5 \\ 10 & \text{if } x \ge 5 \end{cases}$

A) is continuous for real x
B) is discontinuous at $x = 4$ only
C) is discontinuous at $x = 5$ only
D) is discontinuous at $x = 4$ and $x = 5$
E) is discontinuous for $4 < x < 5$

D8. Find the horizontal asymptote of $f(x)=\dfrac{7x^2}{5x-3}$

A) $y = 0$ B) $y = 7/5$ C) $y = -7/3$ D) $y = 7$ E) there is no horizontal asymptote

D9. Find the derivative of $y = 5\ln(2x - 7)$.

A) $\dfrac{5}{\ln(2x-7)}$ B) $\dfrac{5}{2x-7}$ C) $5\ln(2)$ D) $\dfrac{10}{2x-7}$ E) $\dfrac{10}{\ln(2x-7)}$

D10. Find the derivative of $f(x) = 4e^{-3x}$.

A) $-12e^{3x}$ B) $12e^{-3x}$ C) $-12e^{-3x}$ D) $4e^{-3}$ E) $4e^{-3x}$

D11. The derivative of $f(x)=\dfrac{2x-6}{x^2+x}$ is

A) $\dfrac{2x-6}{(x^2+x)^2}$ B) $\dfrac{-2x^2+12x+6}{(x^2+x)^2}$ C) $\dfrac{2}{x^2+x}$ D) $\dfrac{2}{2x+1}$ E) $2(2x - 6)(x^2 + x)$

D12. Find the slope of the line tangent to the graph of the function $f(x) = x^2(2x + 5)$ at $x = 1$.

A) 4 B) 16 C) 14 D) 10 E) 0

D13. Find the slope of the line tangent to $y = \sqrt{(6x+7)^3}$ at $x = 3$.

A) $\dfrac{15}{2}$ B) $9\sqrt{6}$ C) 45 D) $\dfrac{3\sqrt{6}}{2}$ E) $\dfrac{9}{5}$

D14. If $g(x)=\sqrt{x}+\frac{1}{2}x^2$, then $g'(x)$ is

A) $\dfrac{2}{\sqrt{x}}+x$ B) $x+\dfrac{1}{2\sqrt{x}}$ C) $\frac{1}{2}x^{3/2}+x$ D) $2(x+\sqrt{x})$ E) $2x + x^2$

D15. Evaluate $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$ for $f(x) = 4x^2 + 7x$.

A) 1 B) $\dfrac{\Delta x + 14x}{\Delta x}$ C) $7 + 4\Delta x$ D) $8x + 7 + 4\Delta x$ E) $22x + 4(\Delta x)^2 + 7\Delta x$

D16. Find $\lim\limits_{\Delta x \to 0} \dfrac{2x\Delta x + 3(\Delta x)^2 + 7\Delta x}{\Delta x}$.

A) $2x + 7$ B) $2x + 10$ C) $2x$ D) ∞ E) 0

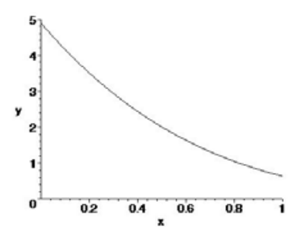
D17. Given $y = x^2 + 4x$, find the value of the differential dy if x changes from 2 to 5.

A) 6 B) 16 C) 24 D) 22 E) -2

D18. (0, 6) (i.e. $x = 0$ and $y = 6$) is the only critical point of $f(x) = 4x^6 + 3x^4 + 6$. You do not have to verify this. Determine what is true of (0, 6).

A) (0, 6) is a relative maximum. B) (0, 6) is a relative minimum C) (0, 6) is a saddle point
D) (0, 6) is not a relative extremum E) no conclusion is possible

D19. On the interval $0 < x < 1$, for the graph shown,



A) $f'(x) > 0$ and $f''(x) > 0$
B) $f'(x) > 0$ and $f''(x) < 0$
C) $f'(x) < 0$ and $f''(x) > 0$
D) $f'(x) < 0$ and $f''(x) < 0$
E) None of the above

D20. The slope of the line tangent to the curve $x^2 + y^2 = 24$ at the point (-4, 3) is

A) 4/3 B) -4/3 C) 3/4 D) -3/4 E) 12

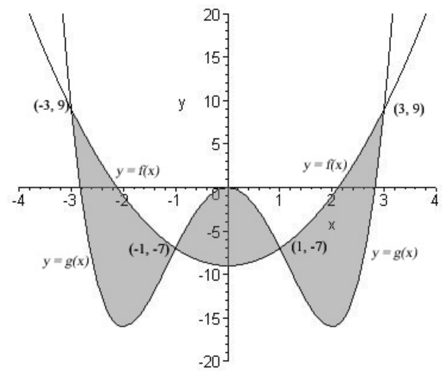
D21. If $y = 3x^3 - 6x + 4$, find dx/dt when $x = 2$ and $dy/dt = 60$.

A) ½ B) 2 C) 4 D) 30 E) 1800

D22. Find the average value of $f(x) = 2x - 5$ on the interval [-1, 3] (i.e. $-1 \le x \le 3$).

A) -3/2 B) 3/2 C) 2 D) -3 E) 3

D23. In the graph on the right the parabola is $f(x) = 2x^2 - 9$ and the function whose graph turns 3 times is $g(x) = x^4 - 8x^2$. The total area of the three shaded regions combined is



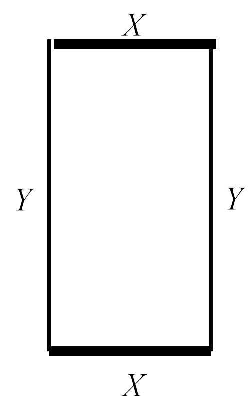
A) $\int_{-3}^{-1}(f(x)-g(x))dx+\int_{-1}^{1}(g(x)-f(x))dx+\int_{1}^{3}(f(x)-g(x))dx$

B) $\int_{-3}^{-1}(g(x)-f(x))dx+\int_{-1}^{1}(f(x)-g(x))dx+\int_{1}^{3}(g(x)-f(x))dx$

C) $\int_{-3}^{3}(f(x)-g(x))dx$ D) $\int_{-3}^{3}(g(x)-f(x))dx$

E) None of the above

D24. Select the correct mathematical formulation of the following problem. A rectangular area must be enclosed as shown. The sides labeled x cost \$12 per foot. The sides labeled y cost \$8 per foot. If the area must equal 900 square feet, what should x and y be in order for the cost to be as small as possible?



A) Minimize xy if $24x + 16y = 900$
B) Minimize xy if $2x + 8y = 900$
C) Minimize $24x + 16y$ if $xy = 900$
D) Minimize $12x + 8y$ if $xy = 900$
E) Minimize $2x + 2y$ if $xy = 900$

D25. For a production level of x units of a commodity, the cost function in dollars is $C = 200x + 4100$. The demand equation is $p = 300 - 0.05x$. What price p will maximize the profit?

A) \$100 B) \$250 C) \$900 D) \$1500 E) \$6000

PART II: CALCULATOR ALLOWED

D26. If $A = \begin{bmatrix} 1 & -1 \\ 4 & 5 \end{bmatrix}$, find the inverse, A^{-1}.

A) $\begin{bmatrix} 5/9 & -1/9 \\ 4/9 & 1/9 \end{bmatrix}$ B) $\begin{bmatrix} 5/9 & 4/9 \\ -1/9 & 1/9 \end{bmatrix}$ C) $\begin{bmatrix} 5/9 & 1/9 \\ -4/9 & 1/9 \end{bmatrix}$ D) $\begin{bmatrix} -1/9 & 1/9 \\ -4/9 & -5/9 \end{bmatrix}$

E) Inverse does not exist

D27. Solve $\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 4 \end{bmatrix}$.

A) $\begin{bmatrix} 1 \\ 7 \\ 4 \end{bmatrix}$ B) $\begin{bmatrix} 3 \\ 11 \\ -15 \end{bmatrix}$ C) $\begin{bmatrix} 1/3 \\ 7/3 \\ 4/3 \end{bmatrix}$ D) $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ E) $\begin{bmatrix} -1 \\ 14/3 \\ -20/3 \end{bmatrix}$

D28. Find $\lim\limits_{x \to 1^+} \dfrac{2+x}{1-x}$

A) ∞ B) -∞ C) -1 D) 2 E) 0

D29. Find the critical numbers for $f(x) = \dfrac{e^x - x^6}{10{,}000}$

A) 0.482 only B) 0.482 and 13.94 C) 1 D) 0.824 only E) 0.824 and 15.494

D30. If x dollars are spent on advertising, then the revenue in dollars is given by $R = \dfrac{100{,}000}{1 + 2^{15-0.005x}}$, where $0 \le x \le 5000$. How much is spent on advertising at the point of diminishing returns (point of inflection)?

A) $10,576 B) $100,000 C) $50,086.60 D) $3000 E) There is no point of diminishing returns

D31. Given the demand equation $p = -3x + 30$ and the supply equation $p = 2x + 5$, find the producer surplus.

A) 50 B) 25 C) -37.5 D) 112.5 E) 62.5

D32. Evaluate $\displaystyle\int_4^8 \frac{3}{\sqrt{2x-7}}\,dx$

A) Non-real Result B) -4 C) -0.821122 D) -0.60264 E) 6

D33. The total profit of a company since it started business is given by $P(t) = 1500t - 3\sqrt{t} - 50{,}000$, where t is the number of months since the company started business. Find the average rate of change in profit with respect to time between 3 months after the company started and 7 months after the company started.

A) 5997.26 B) -42,506.6 C) 2998.57 D) 1499.28 E) 1499.31

D34. The demand function for a product is $p = 100 - 0.1x\ ln(x)$ where x is the number of units of the product sold and p is the price in dollars. Find the value of x for which the marginal revenue is one dollar per unit.

A) 97.45 B) 0.000017 C) 188.9 D) 190.5 only E) 0.01 and 190.5

D35. $x = -1/3 \approx -0.333$ is a critical number for $f(x) = e^x \sqrt[3]{x}$. The critical point (-0.333, -0.497) is

A) a relative minimum B) a relative maximum C) neither a relative maximum nor minimum
D) a point of inflection E) a saddle point

SAMPLE EXAM E

PART I: NO CALCULATOR ALLOWED

E1. The critical numbers for $y = x^4 - 2x^2$ are

A) $x = 0$ only B) $x = -1, 0, 1$ C) $x = -1, 1$ only D) $x = -2$ only E) $x = -4, 4$ only

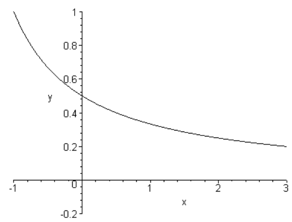
E2. If $y = 2x^2 + 5x + 1$, find the value of the differential dy corresponding to a change in x of $dx = 0.4$ when $x = 5$.

A) 76.4 B) 25.4 C) 25 D) 10 E) 6

E3. (0, 2) (i.e. $x = 0$ and $y = 2$) is the only critical point of $f(x) = 2 - 5x^4 - 3x^6$. You do not have to verify this. Determine what is true of (0, 2).

A) (0, 2) is a relative maximum. B) (0, 2) is a relative minimum C) (0, 2) is a saddle point
D) (0, 2) is not a relative extremum E) no conclusion is possible

E4. On the interval $0 < x < 2$, for the graph shown,



A) $f(x) > 0$, $f'(x) < 0$ and $f''(x) > 0$
B) $f(x) < 0$, $f'(x) < 0$ and $f''(x) > 0$
C) $f(x) < 0$, $f'(x) > 0$ and $f''(x) > 0$
D) $f(x) < 0$, $f'(x) < 0$ and $f''(x) < 0$
E) $f(x) > 0$, $f'(x) > 0$ and $f''(x) < 0$

E5. If $5x^2 - 3xy + y = 2$, find dy/dx by using implicit differentiation and evaluate dy/dx at the point (0, 2).

A) -2 B) -5/4 C) 0 D) 6 E) Does not exist.

E6. Find the point(s) of inflection (x, y) for the function $f(x) = x^3 - 9x^2 + 15x$

A) (1, 7) only B) (5, -25) only C) (1, 7) and (5, -25) D) (3, -9) E) (3, -12)

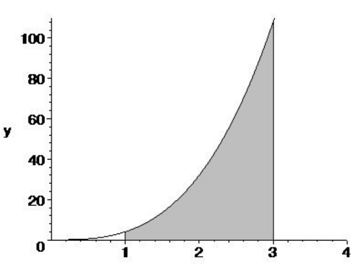
E7. The side of a square is increasing at the rate of 2 feet per minute. Find the rate at which the area is increasing when the side is 7 feet long.

A) 28 ft^2/min B) 14 ft^2/min C) 49 ft^2/min D) 28π ft^2/min E) 49π ft^2/min

E8. If the marginal probability is given by $P'(x) = 300 - 2x$ and the profit that results from producing 20 items is \$3600, find the profit that results from selling 10 items.

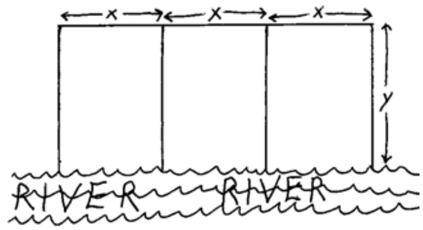
A) \$1800 B) \$900 C) \$150 D) \$4200 E) \$2900

E9. Find the area of the region shown where $y = 4x^3$.



A) 40 B) 82 C) 81 D) 104 E) 80

E10. Select the correct mathematical formulation of the following problem.
A farmer wishes to construct 3 adjacent fields alongside a river as shown. Each field is x feet wide and y feet long. No fence is required along the river, so each field is fenced along 3 sides. The farmer has 900 feet of fence available. What should the dimensions of each field be in order to maximize the area of each field?



A) Maximize $3x + 4y$ if $xy = 900$
B) Maximize $3x + 4y$ if $xy = 300$
C) Maximize xy if $3(x + y) = 900$
D) Maximize xy if $3x + 4y = 900$
E) Maximize xy if $3x + 4y = 300$

E11. Find the maximum profit for the profit function $P(x) = -2x^2 + 10x - 3$.

A) 10 B) 19/2 C) $\frac{5+\sqrt{19}}{2}$ D) 7/4 E) 67/8

E12. If the derivative of $g(x)$ is $g'(x) = (x - 1)^2(x - 2)$ then the function is

A) decreasing on $(-\infty, \infty)$
B) decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$
C) decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$
D) increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$
E) decreasing on $(-\infty, 1)$, increasing on $(1, 2)$ and decreasing on $(2, \infty)$

E13. $C = 0.5x^2 + 15x + 5000$ represents the total cost of producing x units. The production level that minimizes the average cost is

A) $x = 5000$ B) $x = 15$ C) $x = -100$ D) $x = 10{,}000$ E) $x = 100$

E14. The augmented matrix shown represents the solution of a system of equations in the variables (x, y, z, w). What is the solution?

$$\begin{bmatrix} 1 & 0 & -7 & 0 & -3 \\ 0 & 1 & 8 & 0 & 10 \\ 0 & 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A) $(-3, 10, \frac{5}{4}, 9)$
B) $\left(\frac{x+3}{7}, \frac{10-y}{8}, z, 9\right)$
C) $\left(\frac{z+3}{7}, \frac{10-z}{8}, 9, z\right)$
D) $(-3 + 7z, 10 - 8z, z, 9)$
E) No solution

E15. Find the reduced row echelon form that results from using Gauss-Jordan row reduction.

$$4x + 2y = 6$$
$$6x + 3y = 9$$

A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -5 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & 0 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 0 & 0 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

E16. Determine the values of x for which $f(x) = \frac{x-4}{x^2 - 5x + 4}$ is continuous.

A) All real x B) All real $x \neq 4$ C) All real $x \neq 1$ D) All real $x \neq 1, 4$ E) All x in the interval $(0,5)$

E17. Find the horizontal asymptote for the graph of $y = \dfrac{2+x}{1-x}$

A) $y = 0$ B) $y = 1$ C) $y = 3$ D) $y = 2$ E) $y = -1$

E18. The graph of the function $f(x) = \dfrac{x^2+2}{x^2+1}$ has as its vertical asymptote(s)

A) $x = -1$ only B) $x = 1$ only C) $x = 1$ and $x = -1$ D) No vertical asymptotes E) $y = 1$ only

E19. If $y = x^2 \ln x$ (with $x > 0$), then dy/dx equals

A) $x - 2x \ln x$ B) 2 C) $2x \ln x$ D) $x + 2x \ln x$ E) $2x - \ln x$

E20. If $f(x) = e^{3x^2+5x}$, then $f'(x) =$

A) e^{3x^2+5x} B) $e^{3x^2+5x}(6x+5)$ C) e^{6x+5} D) $e^{6x+5}(6x+5)$ E) $e^{6x+5}(3x^2+5x)$

E21. Find $f'(-1)$ for $f(x) = \dfrac{x^3 - 2x^2 + x - 1}{x}$

A) 5 B) -5 C) 3 D) -3 E) -1

E22. Find the value of the derivative of $y = 2x(4x - 10)^3$ at $x = 3$.

A) 288 B) 88 C) 304 D) 96 E) 24

E23. Find the derivative of $f(x) = \sqrt{x^2 - 2x + 3}$.

A) $\sqrt{2x-2}$ B) $\dfrac{1}{2\sqrt{x^2-2x+3}}$ C) $\dfrac{1}{2\sqrt{2x-2}}$ D) $\dfrac{x-1}{\sqrt{x^2-2x+3}}$ E) $\dfrac{1}{\sqrt{x^2-2x+3}}$

E24. The equation of the line tangent to the curve $y = 3x^2 - 2x + 7$ at the point $(1, 8)$ is

A) $y = 4x^2$ B) $y = 6x$ C) $y = 4x + 4$ D) $y = 0$ E) $y = -4x + 8$

E25. Evaluate $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ for $f(x) = 8x^2 + 2x$.

A) 1 B) $\dfrac{\Delta x + 4x}{\Delta x}$ C) $2 + 8\Delta x$ D) $20x + 8(\Delta x)^2 + 2\Delta x$ E) $16x + 2 + 8\Delta x$

PART II: CALCULATOR ALLOWED

E26 and E27. The weekly profit that results from selling x units of a commodity weekly is given by $P = 50x - 0.003x^2 - 5000$ dollars.

E26. Find the actual change in weekly profit that results from increasing production from 5000 units weekly to 5015 units weekly.

A) \$20.00 B) \$300.00 C) \$299.33 D) \$298.00 E) \$8333.33

E27. Use the marginal weekly profit function to estimate the change in weekly profit that results from increasing production from 5000 units weekly to 5015 units weekly.

A) \$20.00 B) \$300.00 C) \$299.33 D) \$298.00 E) \$8333.33

E28. Determine whether or not $y = \dfrac{x-3}{x^2}$ has a relative maximum. If it has a relative maximum, what is the maximum (y value)?

A) 0 B) 0.083333 C) 0.07 D) 0.079467 E) None exists

E29 and E30. Given $A = \begin{bmatrix} 4 & 3 & 5 \\ 7 & 5 & 4 \\ 2 & 1 & -8 \end{bmatrix}$

E29. Find the inverse of A, A^{-1}.

A) $\begin{bmatrix} -44 & 29 & -13 \\ 64 & -42 & 19 \\ -3 & 2 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 47 & 32 & -8 \\ 71 & 50 & 23 \\ -1 & 3 & 78 \end{bmatrix}$ D) $\begin{bmatrix} 1/4 & 1/3 & 1/5 \\ 1/7 & 1/5 & 1/4 \\ 1/2 & 1 & -1/8 \end{bmatrix}$

E) The inverse does not exist.

E30. Solve $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$

A) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ B) $\begin{bmatrix} -182 \\ 265 \\ -13 \end{bmatrix}$ C) $\begin{bmatrix} -167 \\ 110 \\ -50 \end{bmatrix}$ D) $\begin{bmatrix} 7/6 \\ 187/140 \\ -5/8 \end{bmatrix}$ E) No solution exists.

E31. Given $A = \begin{bmatrix} 2 & -3 \\ 0 & 1/5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$, find A^2B.

A) $\begin{bmatrix} 98 \\ 2/5 \end{bmatrix}$ B) $\begin{bmatrix} -26 \\ 2 \end{bmatrix}$ C) $\begin{bmatrix} 8 & 90 \\ 0 & 2/5 \end{bmatrix}$ D) $\begin{bmatrix} -58 \\ 2/5 \end{bmatrix}$ E) $\begin{bmatrix} 8 & 18 \\ 0 & 2/5 \end{bmatrix}$

E32. Find the absolute extrema of $f(x) = \dfrac{2x}{x^2+1}$ on the closed interval [0, 2].

A) (0, 1) and (1, 3) B) (2, 4/5) and (-1, -1) C) (2, 4/5) and (0, 0) D) (1, 1) and (-1, -1)
E) (1, 1) and (0, 0)

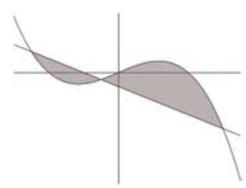
E33. Find $\lim\limits_{\Delta x \to 0} \dfrac{e^{2(x+\Delta x)} - e^{2x}}{\Delta x}$

A) 0 B) e^2 C) e^{2x} D) $2e^{2x}$ E) undefined (does not exist)

E34. Find the average value of $f(x) = (4x - 5)^3$ on the interval [-1, 1] (i.e. $-1 \le x \le 1$).

A) -205 B) 205 C) -410 D) 410 E) -185

E35. A sketch of the graphs of $f(x) = 16x - x^3$ and $g(x) = -15x - 30$ appears on the right. Find the total area of the shaded region.



A) 524.75 B) 332.75 C) 492.25 D) 416.75 E) 399.25

ADDITIONAL PROBLEMS

PART I: NO CALCULATOR ALLOWED

F1. Find all of the points at which the graph of $f(x) = x^4 - 4x + 5$ has horizontal tangent lines.

A) (2,0) only B) (1,0) only C) (1,2) and (-1,10) D) (1,0) and (2,13) E) (1,2) only

F2. Find dy/dx if $xe^y + 1 = xy$

A) 0 B) $\frac{y - e^y}{xe^y - x}$ C) $\frac{y}{e^y - x}$ D) $\frac{e^y}{xe^y - 1}$ E) $\frac{y}{xe^y - x}$

F3. Find $\lim\limits_{x \to -4^+} \frac{x+1}{x+4}$.

A) 1 B) 1/4 C) -3/8 D) ∞ E) -∞

F4. Find the intervals on which $f(x) = x^2 - 2x + 1$ is continuous.

A) (0, 2) (i.e $0 < x < 2$) B) (-∞, ∞) (i.e. all real x) C) (0, 1) (i.e. $0 < x < 1$)
D) (0, ∞) (i.e. $x > 0$) E) (-4, 4) ∪ (4, ∞) (i.e. $-4 < x < 4$ or $x > 4$)

F5. $f(x) = \frac{3x^2}{x^3 + 2}$ has as a horizontal asymptote

A) $y = 1/3$ B) $y = ½$ C) $y = 0$ D) $y = 3$ E) $y = 3/2$

F6. The slope of the line tangent to $f(x) = \sqrt{x} - 1$ at (4, 1) is

A) 16 B) 3 C) 1/4 D) -1/4 E) 1

F7. The derivative of $f(x) = (x - 5x^2)^7$ is

A) $7(x - 5x^2)^6$ B) $7(1 - 10x)^6$ C) $7(1 - 10x)(x - 5x^2)^6$ D) $70x(x - 5x^2)^6$ E) $(1 - 10x)^6$

F8. The radius r of a circle is increasing at the rate of 3 inches per minute.
What is the rate of change in the area when $r = 20$ inches?

A) 6π inch/min B) 400π in²/min C) 40π in²/min D) 60π in²/min E) 120π in²/min

F9. Find <u>all</u> of the critical numbers for $f(x) = 2\sqrt{9 - x^2}$.

A) 0 B) -3, 3 C) 3 D) -3, 0, 3 E) -3

F10. Find the derivative of $f(x) = x^4 e^x$

A) $4x^3e^x$ B) $x^4e^x + 4x^3e^x$ C) $4x^3 + e^x$ D) $x^4 + e^x$ E) $12x^4e^x$

F11. The function $f(x) = \dfrac{1}{x-2} - 3x$ is discontinuous when

A) $x = 0$ B) $x > 0$ C) $x = 2$ D) $x > 2$ E) never

F12. Find <u>ALL</u> of the vertical asymptotes for $y = \dfrac{x^2-9}{x^2-16}$

A) $x = \pm 4$ B) $x = \pm 3$ C) $y = 1$ D) $y = 9/16$ E) $x = \pm 3, \pm 4$

F13. Find the derivative of $f(x) = \dfrac{5x-4}{3x+2}$.

A) $\dfrac{17-15x}{3x+2}$ B) $\dfrac{22}{(3x+2)^2}$ C) $\dfrac{-2}{(3x+2)^2}$ D) $\dfrac{10x+16}{(3x+2)^2}$ E) $\dfrac{5}{3}$

F14. Find the slope of the line tangent to the graph of $f(x) = 2x^{2/3} - 2x + 9$ at the point (8, 1).

A) 11 B) -11/6 C) 1 D) -4/3 E) -8

F15. Determine the x coordinate of the point(s), if any, at which the function $y = \frac{1}{3}x^3 - x^2 - 15x + 6$ has a horizontal tangent.

A) $x = 0$ B) $x = -3, 5$ C) $x = 3, -5$ D) Such points do not exist. E) $x = -15, 1$

F16. If $f(x) = 8x^4 - 10x^2 + 3x - 1$, find the value of $f''(x)$ (the second derivative) at $x = 2$.

A) 219 B) 364 C) 93 D) 376 E) 384

F17. Find $\lim\limits_{x\to\infty} \dfrac{4x-10}{16-2x}$

A) 0 B) Does not exist (undefined) C) 4 D) -10 E) -2

F18. If the demand equation is given by $p = \sqrt{9-x}$, then the marginal revenue is 0 when

A) $x = 9$ B) $x = 2$ C) $x = 6$ D) $x = 3$ E) $x = 5$

F19. For what values of x is the function $f(x)=\frac{x-1}{x^2-x-2}$ not continuous?

A) 1 B) -2 and 1 C) -5, -2 and 1 D) -1 and 2 E) -1, 1 and 2

F20. If $A=\begin{bmatrix}2 & -3\\ 1 & 0\end{bmatrix}$, find A^2.

A) $\begin{bmatrix}4 & 9\\ 1 & 0\end{bmatrix}$ B) $\begin{bmatrix}1 & -6\\ 2 & -3\end{bmatrix}$ C) $\begin{bmatrix}5 & -6\\ -6 & 9\end{bmatrix}$ D) $\begin{bmatrix}4 & -6\\ 2 & 0\end{bmatrix}$ E) Undefined

F21. Perform the matrix operations: $3\begin{bmatrix}2 & -1 & 0\\ 5 & 6 & -3\end{bmatrix}+2\begin{bmatrix}1 & -5 & 2\\ 0 & 3 & 7\end{bmatrix}$

A) $\begin{bmatrix}15 & -30 & 10\\ 25 & 45 & 20\end{bmatrix}$ B) $\begin{bmatrix}8 & 15\\ -13 & 24\\ 4 & 5\end{bmatrix}$ C) $\begin{bmatrix}8 & -6 & 2\\ 15 & 9 & 4\end{bmatrix}$ D) $\begin{bmatrix}8 & -13 & 4\\ 15 & 24 & 5\end{bmatrix}$ E) $\begin{bmatrix}11 & -10 & 2\\ 25 & 33 & -8\end{bmatrix}$

F22. Solve the following system of equations:

$$\begin{aligned} x - y &= 8\\ 2x + y &= 7\\ x - 3y &= 15\end{aligned}$$

A) (10, 2) B) (8, -3, -3) C) There are infinitely many solutions D) No solution exists E) (5, -3)

F23. Find the following product: $\begin{bmatrix}1 & -3 & 5\end{bmatrix}\begin{bmatrix}4 & -4 & 0\\ 1 & 2 & 2\\ 3 & -1 & 0\end{bmatrix}$

A) $\begin{bmatrix}-8\\ 5\\ 6\end{bmatrix}$ B) $\begin{bmatrix}-8 & 5 & 6\end{bmatrix}$ C) $\begin{bmatrix}16 & -15 & -6\end{bmatrix}$ D) $\begin{bmatrix}16\\ -15\\ -6\end{bmatrix}$ E) $\begin{bmatrix}4 & 12 & 0\\ 1 & -6 & 10\\ 3 & 3 & 0\end{bmatrix}$

F24. Find all solutions to the system of equations.

$$\begin{aligned} x + w &= 5\\ y - z &= -1\\ 2x + y + w &= 8\end{aligned}$$

A) (5 - w, -2 + w, -1 + w, w) B) (5, -2, -1, 0) C) (1, -1, -1, 0) D) (4, -1, 0, 1) E) No solution exists

PART II: CALCULATOR ALLOWED

F25. Find $\lim\limits_{x \to 4^+} \frac{x^2}{x^2 - 16}$

A) 0 B) Does not exist (undefined) C) ∞ D) -∞ E) 1

F26. For what values of x is the slope of the line tangent to $y = x^3 + 11x^2 - x + 17$ equal to 0?

A) None B) $\frac{-11 \pm 2\sqrt{31}}{3}$ (-7.37851 or 0.045176) C) 11/17 D) 0 E) $\pm\sqrt{31} = \pm 5.56776$

F27. Given $f(x) = \frac{x^3 - 1}{x^4 + 5x + 17}$ find $f''(1)$.

A) 84 B) 0 C) 529 D) 84/529 E) 1

F28. For which value of x does the graph of the function $f(x) = e^{-x} \ln x$ have a horizontal tangent? Round your answer to the nearest hundredth.

A) 1 B) 1.76 C) 2.81 D) 3.93 E) Never

F29. Find the equation of the straight line tangent to $f(x) = \frac{2x+3}{x-4}$ at the point (3, -9).

A) None exists (false) B) $y = \frac{-11x}{(x-4)^2} + 24$ C) $y = -22x + 57$ D) $y = -11x + 24$ E) $y = -11$

ANSWERS

SAMPLE EXAM A	SAMPLE EXAM B	SAMPLE EXAM C	SAMPLE EXAM D	SAMPLE EXAM E	ADDITIONAL PROBLEMS
PART I	PART I	PART I	PART I	PART I	PART I
A1. D	B1. B	C1. D	D1. E	E1. B	F1. E
A2. A	B2. A	C2. E	D2. A	E2. D	F2. B
A3. D	B3. A	C3. E	D3. D	E3. A	F3. E
A4. D	B4. B	C4. D	D4. A	E4. A	F4. B
A5. D	B5. C	C5. B	D5. C	E5. D	F5. C
A6. B	B6. E	C6. E	D6. C	E6. D	F6. C
A7. A	B7. D	C7. A	D7. A	E7. A	F7. C
A8. C	B8. A	C8. D	D8. E	E8. B	F8. E
A9. C	B9. D	C9. A	D9. D	E9. E	F9. D
A10. D	B10. A	C10. A	D10. C	E10. D	F10. B
A11. C	B11. B	C11. E	D11. B	E11. B	F11. C
A12. B	B12. D	C12. D	D12. B	E12. C	F12. A
A13. A	B13. E	C13. C	D13. C	E13. E	F13. B
A14. B	B14. D	C14. C	D14. B	E14. D	F14. D
A15. B	B15. D	C15. B	D15. D	E15. C	F15. B
A16. C	B16. B	C16. B	D16. A	E16. D	F16. B
A17. C	B17. C	C17. B	D17. C	E17. E	F17. E
A18. E	B18. E	C18. D	D18. B	E18. D	F18. C
A19. D	B19. E	C19. E	D19. C	E19. D	F19. D
A20. A	B20. A	C20. A	D20. A	E20. B	F20. B
A21. A	B21. C	C21. A	D21. B	E21. D	F21. D
A22. D	B22. D	C22. C	D22. D	E22. C	F22. D
A23. E	B23. B	C23. C	D23. A	E23. D	F23. C
A24. B	B24. A	C24. E	D24. C	E24. C	F24. A
A25. A	B25. C	C25. D	D25. B	E25. E	
PART II	PART II	PART II	PART II	PART II	PART II
A26. E	B26. A	C26. E	D26. C	E26. C	F25. C
A27. B	B27. B	C27. B	D27. D	E27. B	F26. B
A28. A	B28. C	C28. C	D28. B	E28. B	F27. D
A29. C	B29. A	C29. A	D29. E	E29. A	F28. B
A30. B	B30. A	C30. D	D30. D	E30. B	F29. D
A31. D	B31. A	C31. C	D31. B	E31. D	
A32. D	B32. A	C32. E	D32. E	E32. E	
A33. D	B33. D	C33. B	D33. E	E33. D	
A34. E	B34. D	C34. E	D34. A	E34. A	
A35. B	B35. C	C35. C	D35. A	E35. A	